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FUNDAMENTAL CONSIDERATIONS OF THE CROSSED-BEAM CORRELATION TECHNIQUE (Final Report)

Prepared under Contract No. NAS 8-11258 by
M. J. Fisher and R. J. Damkevala

IIT RESEARCH INSTITUTE

For

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama

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IIT RESEARCH INSTITUTE
Chicago, Illinois

For

Aero-Astrodynamics Laboratory

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

FUNDAMENTAL CONSIDERATIONS OF THE CROSSED BEAM CORRELATION TECHNIQUE

FOREWORD

The object of this report is to present in one unified document many of the theoretical concepts which form a foundation to the newly developed "Crossed Beam Correlation Technique." It is hoped that the availability of such a document will be beneficial to those wishing to use this technique in offering a convenient source of reference.

Of the many people, both of the IIT Research Institute and George C. Marshall Space Flight Center, who have contributed to this work, the authors have particular pleasure in acknowledging Dr. F. Krause, who has played a major role in the conceptual developments presented here.

ABSTRACT

The desire and necessity to measure turbulence in regions inaccessible to standard probes as diverse as the base recirculation region of rocket launch vehicles and in the atmosphere has led to the development of a number of optical techniques. One of these, "The Crossed Beam Correlation Technique" is the subject of this report, our object being to summarize the conceptual details on which this method is based.

The "crossed beam correlation technique" avoids the necessity of inserting solid probes into the flow field. Instead, two beams of radiation are employed which traverse the entire flow field in two mutually perpendicular directions. The radiation employed is chosen so that it is either absorbed or scattered by a flow constituent. Thus, turbulence induced fluctuations of either the thermo-dynamic properties or concentrations of the chosen property result in fluctuations in detected light intensity. Each beam alone reflects only an integral of the fluctuation occurring along its entire path. However, it is shown that cross correlation of the two detected signals eliminates much of the integration yielding local turbulent properties instead. Specifically it is shown that the local value of the intensity of the fluctuations, the integral scales of turbulence, the convection speeds, turbulent spectrum and moving axes time scales can be obtained using this method.

Previously experimental results in subsonic (6) flow have been published which confirm the theoretical predictions, and are shown to be in good agreement with previous data obtained from more conventional techniques.

It is also shown that the method offers a distinct advantage over probe techniques in the estimation of turbulence forcing functions, while the hitherto elusive three-dimensional spectrum function can be obtained from anisotropic flow.

ACKNOWLEDGEMENTS

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The following support received from the Marshall Space Flight Center is also gratefully acknowledged.

(a) The design and manufacture of the rotatable platform on which the crossed beam system is mounted.

(b) The manufacture of models for the wind tunnel tests.

(c) The design and assembly of the multiplex tape recorder system.

(d) The provision of an analog time delay correlator.

(e) The provision and operation of the MSFC 14 in. wind tunnel and Thermo-Acoustic Jet Facility.

(f) The provision of magnetic tapes, analog to digital conversion facilities, and computer codes for statistical data reduction.

The manpower necessary to operate the test facilities, to set up computer input specifications, to carry out analog to digital conversion and the computer time for data reduction was also supplied by MSFC.

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The government personnel responsible for this support were Mr. R. Felix, Mr. I. Simon, Mr. B. Belew, Mr. J. Heaman, Mr. J. Jones, Mr. J. Curet, and Mr. H. Newberry. Their significant contributions are gratefully acknowledged together with the interest and encouragement supplied by Dr. Geissler and Mr. Dahm.

Finally, special mention should be made of Dr. F. Krause and Mr. J. Johnston, R-AERO-AF both of whom made many significant technical contributions to this work. The conceptual developments, undertaken by Dr. Krause, have in particular proved to be of great value.

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CHAPTER 1

GENERAL INTRODUCTION

The purpose of this report is to summarize and document the initial developmental studies performed in the establishment of the crossed beam correlation technique. At the outset of this work, in June 1964, it was already obvious that a technique for the experimental determination of local turbulent properties of flows around rocket launch vehicles was urgently required. The statistical properties of these fluctuations are required as input to a wide range of structural excitation problems as well as in the prediction of heat, mass and momentum fluxes. Practical considerations include the analysis of control systems, structural failures, base heating and the prediction of acoustic environments in which the vehicle must operate. The measurement of turbulent fluctuations therefore represents one of the major problems in the development of launch vehicles (1).

It was also equally obvious that well established techniques were inadequate for use in the supersonic or hot burning flows typical of these vehicles. Admittedly the choice was not large. The hot wire anemometer appears to be virtually the only instrument to find general adoption for this type of measurement. However, the use of any probe instrument of this type is difficult in supersonic flows due to the generation of shock waves by the probe support which are capable of distorting intolerably the very fluctuations one wishes to measure. In hot burning flows the temperature environment is usually sufficient to destroy the probe.

An awareness of these problems suggests the use of optical techniques. The major disadvantage of standard optical methods, such as Schlieren, shadowgraph or interferometry, is that the measured output depends on an integral of the flow properties along the entire light path. This must normally extend through the entire test section while the technique required should give information on the local conditions existing at some point within this test section.

Local fluctuation measurements, using an optical technique, have been made successfully using a viewing technique (2). The basis of such a technique is to focus the image of a powerful light source at the point of interest in the flow. This image is then viewed at an angle to the optical system producing it. In this way the detector system collects only that light which is scattered from the point of interest, and the measured intensity can then be related to the number density of scattering particles contained in the viewed volume. Since this method has most in common, with respect to the measured quantities, with the crossed beam technique, a comparative discussion is presented later in this report. For a comparative discussion of other optical methods, the reader is referred to (3).

As we shall see later, the largest single problem associated with the use of viewing techniques is to ensure that fluctuations of the detected light intensity are truly associated with changes in the number density of scattering centers within the viewed volume. This arises due to the fact that light is also scattered at all points between this position and the source. Thus, fluctuations of number density at any point along this path will cause the light available for scattering at the investigated point to vary. This introduces fluctuations in the scattered radiation detected which is not the result of local turbulence. In principle, a solution to this problem is offered if the scattering process is very weak so that only a small percentage of the incident radiation is scattered from the path of the incident beam. However, because it is the same process which is responsible for scattering on this path and at the point of interest, this reduces the amount scattered into the detector system to a small percentage of the incident light. Thus, extremely powerful sources or low noise detectors are required if small fluctuations are to be reliably detected. It is reported in (2) that the range of tracer concentration between that which is sufficient to yield detectable signals and that which creates erroneous scattering is rather narrow.

These problems are largely overcome using the crossed beam correlation technique, the principle of which is shown in Fig. 1. Two collimated beams of radiation are arranged to intersect at the point to be investigated. The radiation employed is chosen so that it is partially absorbed or scattered by a constituent of the flow. Thus, turbulence induced fluctuations of either the thermodynamic properties or concentration of the chosen constituent are reflected as fluctuations of intensity of the resultant beams. Each beam alone, of course, reflects only an integral of the flow properties along its entire path. However, it will be shown in Chapter 2 that cross correlation of the two resultant beam intensities eliminates much of the integration, yielding local turbulent information instead. It is shown that using this method, local estimates of turbulent intensity, eddy scales, spectra, convection speeds and eddy lifetimes can be obtained.

Since the inception of this work in June 1964, it has become apparent that the crossed beam method has a number of basic advantages to offer when compared with other methods. In comparison with the viewing technique discussed above, it is apparent that the use of a tracer is not prerequisite to the application of the crossed beam method. Suitable choice of the wavelength, and wavelength interval, of the radiation employed will permit the required fluctuations of detected light intensity to be created by absorption due to a species naturally present in the flow. Further selectivity would also permit a particular property (i.e., pressure, temperature, or concentration) of that specie to be chosen for study. We are not aware of another technique which offers this degree of flexibility. Even in the event that a tracing method is chosen, and experience has shown this is often experimentally convenient when it is the kinematic as opposed to the dynamic properties of the flow which are of interest, the stringent restrictions on tracer concentration for the viewing technique no longer apply for the crossed beam method.

Further, when considering the application of many optical techniques to turbulence measurement, detector noise problems will often be an important feature in determining feasibility. This arises from two requirements. First, to obtain sufficient spatial (i.e. wave number) resolution of the flow field it is necessary to limit the field of view of the detector thereby reducing the amount of light available. Secondly, the electronic bandpass of the detector must be sufficient to pass all the turbulent information of interest. Thus, while restriction of the field of view limits the amplitude of the signal to be detected, the necessary increase of the detector bandpass increases the noise power generated. This is particularly true in a shot noise limited case where the noise power is directly proportional to the electronic bandpass. Although each independent arm of a crossed beam system is subject to these problems they are largely eliminated by the subsequent cross correlation procedure. The basic output of the crossed beam system is the covariance of the detected signals. Thus in the normal circumstance that the noise of the two detectors is mutually random, the noise will not effect the value of the covariance. In principle, therefore, any degree of noise can be tolerated in a crossed beam experiment. In practice, of course, there is a practical limitation depending on the accuracy to which the required covariance can be measured. However, experience has shown, and we shall demonstrate later, that equal power associated with the genuine signal and the noise is perfectly tolerable in a crossed beam system. This is in contrast to the more normal necessity of a ratio of at least 10^2 and preferably 10^4 for many methods. The importance of this feature is thrown into sharp relief when one considers the fact that in a shot noise limited situation, a brightness increase by a factor of 10^4 in the light source would be required to obtain the signal to noise ratio of 10^4 for a single system over that which would provide the unity signal to noise ratio required for the crossed beam system.

Turning finally to a comparison of the crossed beam method with techniques which employ point probes such as the hot wire anemometer, certain minor differences are apparent. Primarily, as we shall see below, the technique does not strictly yeild pointwise information. The measured output yields an integral over a correlation area around the beam intersection point. This factor is perhaps the one primary disadvantage of the method to the extent that it puts the spatial resolution of the method beyond the control of the experimenter. However, both our theoretical considerations and experiments indicate that the weighting of contributions to this integral is such that a most acceptable approximation to local values is obtained. Further theoretical work (4) has indicated that this integrating feature of the crossed beam system can, in fact, be advantageous. The statistical properties of turbulent fluctuations required as input for a broad range of calculations, such as structural excitation and aerodynamic noise generation, take the form of either area or volume integrals of space-time correlation functions. While the exact evaluation of these integrals from pointwise data is extremely laborious (and often inaccurate), it is shown in (4) that they can often be obtained from a single measurement using the crossed beam system or at worst from relatively few measurements. Finally, in this context it is also demonstrated in (4) that the integrating feature of this system will permit the direct measurement of the three dimensional spectrum function as opposed to the one dimensional function obtained from point probes (5). The use of point probes has, to date, precluded the estimation of this valuable function except in the rather academic case of pure isotropic turbulence where a unique relation exists between the one and three dimensional spectrum function. Preliminary work indicates that the ability to measure the three dimensional function in anisotropic flows may well further our understanding of the basic structure of turbulent shear flows.

Finally in closing this introduction, it is to be emphasized that many of the concepts presented apply to situations in which the direction of the <u>mean</u> flow velocity is known and the beams of radiation can, to a first approximation, be placed in a plane perpendicular to that direction. When this condition does not apply, for example in the atmosphere, more complicated analyses apply, which are beyond the scope of this report.

CHAPTER 2

THEORETICAL DEVELOPMENT OF THE CROSSED BEAM METHOD

The basic concept of the crossed-beam correlation technique can best be described with reference to Fig. 1. A region of turbulent flow is supposed to be contained within the broken line, this flow being convected in a direction perpendicular to the plane of the diagram. Two optical systems are now arranged, which pass collimated beams of radiation across the flow in two mutually perpendicular directions so that they intersect at the point to be investigated. The wavelength of this radiation is arranged so that it is partially, but not completely, absorbed by one or more species of the flow. Thus turbulent fluctuations of the concentration or density of the chosen species will be reflected in changes of light intensity observed at the detectors. In common with the optical methods mentioned previously, each beam alone reflects only an integral of the appropriate fluctuation along its entire path length. However, as is shown below, the covariance existing between the signals at the two detectors does yield local information.

The retrieval of this local information can be explained intuitively as follows: The instantaneous signal at each detector represents the sum of all fluctuations occurring along its path at a particular time. The fluctuations can be categorized into two groups. First, those fluctuations which occurred sufficiently close to the beam intersection point to introduce a related (or correlated) fluctuation in both beams. The remainder which occur at a sufficient distance from this point are uncorrelated and thus introduce unrelated effects on the beam intensities. If, subsequently, the covariance (time averaged product) of the two detected signals is estimated, those portions of the signal created by the unrelated (uncorrelated) flow fluctuations will yield an average value of zero. The related or correlated fluctuations on the other hand yield a finite averaged product. Thus, the measured covariance is a function only of those fluctuations which occur within the correlated area surrounding the beam intersection point. It remains, of course, to demonstrate that this covariance can be used as a measure of required turbulence parameters.

ANALYTICAL DESCRIPTION

In order to demonstrate that the covariance of the two detected fluctuations does, in practice, yield a measure of required turbulent properties, it will be convenient to introduce the following coordinate system. The point of beam intersection has coordinates (x,y,z) where the y and z axes are oriented along the directions of the beams S_1D_1 and S_2D_2 respectively. Distances from the point of beam intersection are denoted by ξ, η, and ζ in the x, y, and z directions, respectively.

Considering first the beam S_1D_1, the intensity recorded at detector D_1 at time t can be written

$$I_y(t) = I_o e^{-\int K(x,y+\eta,z,t)d\eta} \tag{2.1}$$

where I_o denotes the intensity of the initial beam and K is the appropriate extinction coefficient. The term 'extinction' is employed here to cover a number of possible methods of achieving the required beam attenuation. For example, pure absorption by a flow constituent could be employed, while scattering by particulate matter in the flow offers a second possibility. The term extinction is used here to cover either phenomena or a combination of both. For a more complete discussion of both the generalized and special definitions of the extinction coefficient the reader is referred to [3]. It should also be pointed out that the value of K will additionally be a function of the wavelength of the radiation employed. However, since this dependence does not affect the present discussion, it will not be explicitly shown.

Whatever the actual mechanism of extinction, it should obviously be chosen so that the value of the coefficient depends on a required flow property. Thus since in a turbulent flow the flow properties are a function of both position and time, the extinction coefficient will be similarly dependent. Throughout this report, for the sake of generality, we shall refer to fluctuations of the extinction coefficient. However, since these changes will always reflect fluctuations of a flow property, statistical properties of the flow will be considered as synonymous with statistical properties of the extinction coefficient.

Returning to Eq. (2.1), we can write the instantaneous extinction coefficient as the sum of its time averaged mean value $\langle K(x,y+\eta,z)\rangle$ and a fluctuation relative to this value $k(x,y+\eta,z,t)$. Then

$$I_y(t) = I_o e^{-\int\langle K(x,y+\eta,z)\rangle d\eta} e^{-\int k(x,y+\eta,z,t)d\eta} \tag{2.2}$$

If the extinction process is now arranged so that the integral of the fluctuations, i.e.,

$$\int k(x,y+\eta,z,t)\, d\eta$$

is sufficiently small to permit linearization of that portion of the exponential, Eq. (2.2) can be written

$$I_y(t) = I_o e^{-\int\langle K(x,y+\eta,z)\rangle d\eta}[1 - \int k(x,y+\eta,z,t)d\eta] \tag{2.3}$$

It should be made clear this is not an assumption which restricts the method to small fluctuations. First, the integral in question represents a sum of a number of statistically independent events, which will tend to reduce its value. Secondly, it is shown below that if the integral of the fluctuations is of order or less than 10 percent of the mean integrated value, an optimum value for the mean attenuation is given by

$$\int \langle K(x,y+\eta,z)\rangle \, d\eta = 1$$

For this magnitude of fluctuation, the linearization would be acceptably accurate. In the event that larger fluctuations relative to the mean value are experienced, it would be both desirable and acceptable to reduce the mean absorption so that linearization is still possible.

If the signal at the detector is now written, in turn, as the sum of its time averaged value $\langle I_y \rangle$ and a fluctuation $i_y(t)$ relative to this value, it is easily shown that

$$i_y(t) = -\langle I_y \rangle \int k(x,y+\eta,z,t)\,d\eta \qquad (2.4)$$

Thus, within the limits of the above discussion, we obtain the expected result that the fluctuation at the detector is proportional to the instantaneous integral of the fluctuations along the entire light path.*

Considering next the beam S_2D_2, a similar result can be written down by inspection, namely

$$i_z(t) = -\langle I_z \rangle \int k(x,y,z+\zeta,t)\,d\zeta \qquad (2.4a)$$

If we now take these two fluctuating signals and measure their time averaged product or covariance, we can define a quantity $G(x,y,z)$ where

$$G(x,y,z) \equiv \frac{1}{T}\int_0^T i_y(t)\,i_z(t)\,dt \qquad (2.5)$$

---

*In the event that a situation arises in which fluctuations which are a high percentage of the mean value are to be measured and where a weak extinction process is not available, a result of the form of Eq. (2.4) can be obtained by introducing a logarithmic response amplifier at the output of the detector $i_y(t)$, then representing the output of this additional amplifier.

where T denoted a period of integration, which is of sufficient length to yield a statistically stationary value of $G(x,y,z)$. Substituting for $i_y(t)$ and $i_z(t)$ from Eqs. (2.4) and (2.4a) respectively and reversing the order of spatial and temporal integration, Eq. (2.5) can be written

$$G(x,y,z) = \langle I_y \rangle \langle I_z \rangle \int_\eta \int_\zeta \frac{1}{T} \int_0^T k(x,y+\eta,z,t)k(x,y,z+\zeta,t)dtd\zeta d\eta \tag{2.6}$$

To summarize, taking the fluctuating portions of the two detected signals and measuring their covariance, we obtain the result represented by Eq. (2.6).

SPATIAL RESOLUTION OF COVARIANCES AND MEAN SQUARE VALUES

We can most conveniently understand the significance of this result by considering initially the temporal integration, that is,

$$\frac{1}{T} \int_0^T k(x,y + \eta,z,t)\ k(x,y,z + \zeta,t)dt \tag{2.7}$$

alone. This term, clearly represents the covariance of the fluctuations at the points $(x,y + \eta,z,t)$ and $(x,y,z + \zeta,t)$. If one or both of these points are sufficiently far from the beam intersection point, the flucutations will be mutually random and the resulting covariance will be zero. In fact, only those points contained within the correlated area around the beam intersection point will contribute to the measured value of $G(x,y,z)$. Thus, although formally the limits of spatial integration in Eq. (2.6) extend from source to detector, the value of $G(x,y,z)$ is not changed if these limits are replaced by those corresponding to the limits of the locally correlated area. In this way, therefore, the measured quantity reflects only local turbulent information.

Rewriting the covariance in Eq. (2.6) as the product of the rms intensities at the points considered and a space correlation coefficient $R(x,y+\eta,z+\zeta)$, we obtain*

$$G(x,y,z) = \langle I_y\rangle\langle I_z\rangle \int_\eta\int_\zeta \left\{\overline{k^2(x,y+\eta,z,t)}\ \overline{k^2(x,y,z+\zeta,t)}\right\}^{1/2} R(x,y+\eta,z+\zeta)\,d\zeta\,d\eta \tag{2.8}$$

If the intensity does not vary appreciably over the correlated area (i.e., the region for which the correlation coefficient is finite) then the measured quantity, $G(x,y,z)$, is proportional to this intensity and an area which, by analogy to the familiar concept of an integral length scale, we shall term the integral correlation area, i.e.,

$$\frac{G(x,y,z)}{\langle I_y\rangle\langle I_z\rangle} = \overline{k^2(x,y,z)}\ A(x,y,z) \tag{2.8a}$$

where

$$A(x,y,z) \equiv \int_\eta\int_\zeta R(x,y+\eta,z+\zeta)\ d\zeta\,d\eta \tag{2.8b}$$

In the more likely event that the intensity of the fluctuations does vary over the correlated area, the situation does become somewhat more complicated but, as we shall now proceed to demonstrate, there are a number of reasons for supposing that Eq. (2.8a) is still applicable.

Let us define two functions $f(\eta)$ and $g(\zeta)$ such that

$$\overline{k^2(x,y+\eta,z,t)}^{\,1/2} \equiv \overline{k^2(x,y,z,t)}^{\,1/2} + f(\eta) \tag{2.9a}$$

and

$$\overline{k^2(x,y,z+\zeta,t)}^{\,1/2} \equiv \overline{k^2(x,y,z,t)}^{\,1/2} + g(\zeta) \tag{2.9b}$$

---

*Overbars denote time averaged values.

Basically this involves no more than an expansion of the functional dependence of the rms levels of the fluctuations in Taylor series around the beam intersection point, $f(\eta)$ and $g(\zeta)$ representing the sum of all terms after the first.

Substituting these expressions into Eq. (2.8) we obtain

$$\frac{G(x,y,z)}{\langle I_y\rangle\langle I_z\rangle} = \overline{k^2(x,y,z,t)}\ A(x,y,z)$$
$$+ \overline{k^2(x,y,z,t)}^{1/2} \int_\eta\int_\zeta f(\eta)\ R(x,y+\eta,z+\zeta)\ d\zeta d\eta$$
$$+ \overline{k^2(x,y,z,t)}^{1/2} \int_\eta\int_\zeta g(\zeta)\ R(x,y+\eta,z+\zeta)\ d\zeta d\eta$$
$$+ \int_\eta\int_\zeta f(\eta)\ g(\zeta)\ R(x,y+\eta,z+\zeta)\ d\zeta d\eta \qquad (2.9c)$$

Comparing this equation with (2.8a) we find that the first term on the rhs is that which is obtained when no variation of the rms levels with position is encountered. Thus, the remaining three terms represent the error incurred due to such spatial variations. However, the following arguments would indicate that the magnitude of these terms will normally be small.

Consider first a typical integrand such as

$$f(\eta)\ R(x,y+\eta,z+\zeta)$$

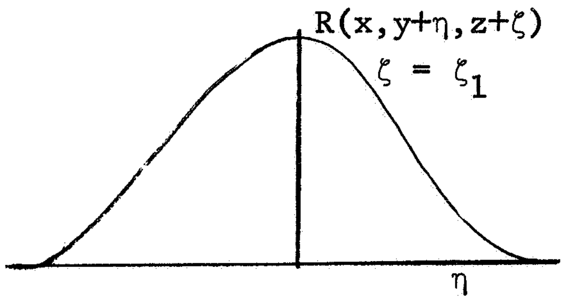
The space correlation coefficient, $R(x,y+\eta,z+\zeta)$ will obtain its maximum value, unity, for $\eta = \zeta = 0$ and will then decrease progressively as either η or ζ increases, that is as the points considered become more remote from each other. Conversely $f(\eta)$, for example, is clearly equal to zero when $\eta = 0$ and in the normal case would increase with increasing η. Thus as one term of the product increases the other decreases and the product itself tends to remain small, finally approaching zero as the points considered become sufficiently remote that no correlation between the fluctuations exists.

Furthermore, irrespective of the magnitude of the integrands, the probable functional forms of $f(\eta)$ and $R(x,y+\eta,z+\zeta)$ would also suggest that considerable cancellation is to be expected in performing the subsequent space integrations.

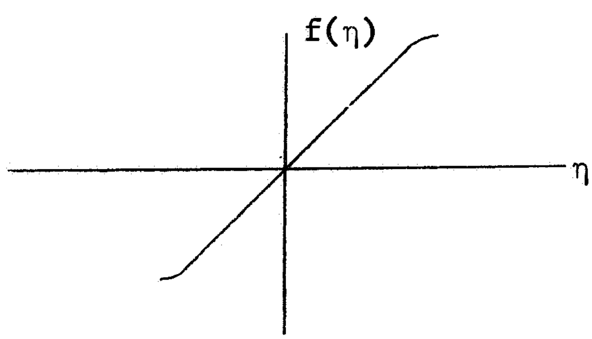
Consider, for example, the second term on the rhs of Eq. (2.9c); namely,

$$\overline{k^2(x,y,z,t)} \int_\zeta\int_\eta f(\eta)\; R(x,y+\eta,z+\zeta)\; d\eta d\zeta$$

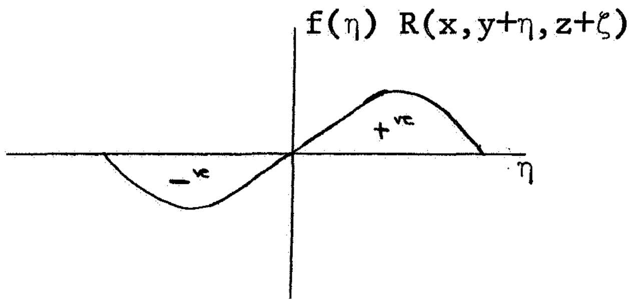
The variation of the space correlation coefficient with η for a given value of ζ will be of the general form shown below.



Conversely in a flow region where the rms level of the fluctuation is, for example, increasing $f(\eta)$ would have the form



resulting in a variation of the product of the schematic form



Thus in performing the required integration over all values of η, considerable cancellation of the area below the axis with that above it will result and the resulting integral will therefore be small.

In fact, in the case that the space correlation coefficient is an even function and $f(\eta)$ is an odd function of η, then the value of the integral under consideration is precisely zero. Perhaps the most obvious case of such an odd functional dependence would be to assume that the variation of rms level is linear over the limited range of flow defined by the correlation area. However, it is emphasized that, although such an assumption would often appear reasonable, it is over-restrictive and any odd functional dependence will suffice.

Clearly similar arguments can be applied to the remaining two terms on the rhs of Eq. (2.9c) and these are similarly expected to be small.

The situation not covered by the foregoing discussion is that which would occur when the beam intersection point is close to a turning point (i.e., either a maximum or minimum) of the rms intensity distribution. In fact, at the turning point the functional dependence of $f(\eta)$, for example, would tend to be even rather than odd as required. Thus, there will be some tendency to underestimate maxima and overestimate minima. However, once again it is anticipated that this error will not be large. Close to such a turning point a function such as $\overline{k^2(x,y+\eta,z,t)}^{1/2}$ will tend to vary comparatively slowly with η.

Thus for small η values, when the space correlation coefficient is large, $f(\eta)$ will be small, while at larger η values where $f(\eta)$ might be more significant, the space correlation coefficient will in its turn have decreased to small values. Thus, the values of their product which determine the magnitude of the "error terms" of Eq. (2.9c) will again tend to be small.

In summary therefore although a variation in the rms level of the extinction coefficient fluctuations across the correlated region leads in principle to the rather complicated situation expressed by Eq. (2.9c) it appears in practice that only the first term on the rhs of this equation will normally be of significance. That is, we may write

$$\frac{G(x,y,z)}{\langle I_y \rangle \langle I_z \rangle} = \overline{k^2(x,y,z,t)}\, A(x,y,z) \tag{2.8a}$$

as a quite general relationship between the measurable quantities on the lhs and the required local flow properties of the rhs of this expression.

One further feature of this result also justifies mention at this time. The quantity $A(x,y,z)$ is by definition, Eq. (2.8b), an integral over space correlation coefficients and thus its value is determined by the scale of the turbulent fluctuations. Previous measurements, such as those reported in [11] for example, indicate that the scales of the turbulent fluctuations are principally a function of position in the streamwise direction and are essentially independent of position in planes perpendicular to that direction. Thus for our present situation $A(x,y,z)$ is expected to be principally a function of 'x' and relatively independent of the beam intersection point in the given y-z plane.

To this degree of approximation therefore, measurements of $G(z,y,z)$, $\langle I_y \rangle$ and $\langle I_z \rangle$ are sufficient to yield a relative intensity profile (i.e., the variation of $\overline{k^2(x,y,z,t)}$) across a chosen cross section of the flow field.

TWO POINT SPACE TIME CORRELATIONS

In order to demonstrate other turbulent properties which can be measured using this technique, let us consider the result obtained when one beam is displaced a distance ξ in the streamwise direction, while in addition a time delay, τ, is introduced between the detected signals prior to estimating their time averaged cross product. This situation is shown schematically in Fig. 2. Denoting the resulting cross correlation by $G(x+\xi,y,z,\tau)$, the result can be written down by inspection of Eq. (2.6).

$$G(x+\xi,y,z,\tau) = \langle I_y\rangle\langle I_z\rangle \int_\eta\int_\zeta \frac{1}{T}\int_0^T k(x,y+\eta,z,t)k(x+\xi,y,z+\zeta,t+\tau)dt\,d\zeta\,d\eta \qquad (2.10)$$

The interpretation of any one term within the double space integral is again straightforward. It represents the space-time covariance for the points $(x,y+\eta,z)$ and $(x+\xi,y,z+\zeta)$ for the value of time delay τ. Therefore finite contributions to the double space integral will be obtained only from those pairs of points which experience a common flucutation. These points will be restricted to a local region around the line of minimum beam separation, where a fluctuation incident on the upstream beam subsequently passes through the downstream beam.

Rewriting Eq. (2.10) in a form similar to Eq. (2.8), we obtain

$$G(x+\xi,y,z,\tau) = \langle I_y\rangle\langle I_z\rangle \int_\eta\int_\zeta \left\{\overline{k^2(x,y+\eta,z,t)}\,\overline{k^2(x+\xi,y,z+\zeta,t)}\right\}^{1/2} R(x+\xi,y+\eta,z+\zeta,\tau)\,d\zeta\,d\eta \qquad (2.11)$$

where $R(x+\xi,y+\eta,z+\zeta,\tau)$ is the appropriate space time correlation coefficient.

If it is assumed that over the local range of η and ζ for which this correlation coefficient is finite the convective flow properties are relatively independent of η and ζ. we can use a separation of variable assumption to obtain

$$R(x+\xi, y+\eta, z+\zeta, \tau) = R(x, y+\eta, z+\zeta)\ r(\xi, \tau) \tag{2.12}$$

Here $r(\zeta, \tau)$ is the space time correlation coefficient which would be measured by two point probes located at the points A and B respectively in Fig. 2. $R(x, y+\eta, z+\zeta)$ is a weighting factor which decreases as the value of η or ζ increases.

Substituting expression (2.12) into Eq. (2.11), the measured cross correlation becomes

$$G(x+\xi, y, z, \tau) = r(\xi, \tau) \langle I_y \rangle \langle I_z \rangle \int_\eta \int_\zeta \left\{ \overline{k^2(x, y+\eta, z, t)}\ \overline{k^2(x+\xi, y, z+\zeta, t)} \right\}^{1/2} \cdot R(x, y+\eta, z+\zeta) d\zeta d\eta \tag{2.13}$$

A comparison with Eq. (2.8) indicates that to a useful degree of approximation $r(\xi, \tau)$, the required space time correlation coefficient is the ratio of two measurable quantities, i.e.,

$$r(\xi, \tau) = \frac{G(x+\xi, y, z, \tau)}{G(x, y, z)} \tag{2.14}$$

If the space time correlation coefficient is measured over a representative range of both ξ and τ the following properties of the turbulent flow field can be obtained.

(a) The space correlation coefficient $r(\xi, 0)$ from which the integral scale of turbulence in the streamwise direction is obtained by integration over all ξ.

(b) The auto-correlation coefficient, $r(0, \tau)$ which yields the local turbulent spectrum by Fourier transformation.

(c) The velocity of convection obtained from the time delay at which a particular cross-correlation curve exhibits a maximum value.

(d) The moving axes auto-correlation which is the envelope of a series of such cross correlation curves.

(e) The eddy lifetime, which may be defined as the time delay for which the moving axes auto-correlation falls to 1/e of its initial value.

The remaining parameter which is needed to define the statistical properties of the turbulent field is, of course, the integral turbulent scale in directions perpendicular to the mean flow direction. Not only are these important characteristics of the turbulence, but in addition they are required to obtain the amplitude of the extinction coefficient fluctuations from the measured parameters shown in Eq. (2.9). In principle, they could be obtained by re-orienting one beam along the flow direction and using the method employed for obtaining the streamwise scale. However, the problems of locating the source and detector to obtain such an orientation are formidable in many flow situations. Thus an alternative method is proposed and discussed below which eliminates any necessity for beam re-orientation.

ESTIMATION OF RADIAL TURBULENT SCALES

The method which has been successfully employed for the estimation of the required radial integral scales of turbulence can best be explained with the aid of Fig. 1. The plane of the diagram represents the plane of the flow containing the intersecting beams and is perpendicular to the flow direction. It has been shown previously that the instantaneous fluctuation (relative to its mean value) of intensity at detector 1 is given by

$$i_y(t) = - \langle I_y \rangle \int k(x,y+\eta,z,t)\, d\eta \qquad (2.4)$$

The time averaged mean square values of the fluctuations at this detector can be most conveniently written in terms of a dummy variable ϵ in the form

$$\overline{i_y^2(t)} = \langle I_y \rangle^2 \int_\eta \int_\epsilon \overline{k(x,y+\eta,z,t)\ k(x,y+\eta+\epsilon,z,t)}\ d\epsilon d\eta \qquad (2.15)$$

The integrand, which represents the covariance of the fluctuations at two points on the line of sight of the detector, can in turn be written in terms of the mean square level of the fluctuations at the points $(x,y+\eta,z,t)$ and a space correlation coefficient.

It is at this point that we must introduce the only new assumption of this analysis; namely, that the scales of turbulence are independent of radial location in the flow. Evidence that such an assumption is close to the truth is borne out by previous measurement in subsonic flows. With such an assertion the space correlation coefficient introduced above becomes only a function of the separation of the points considered (i.e., values of ϵ) and independent of their location. Then Eq. (2.15) can be written

$$\overline{i_y^2(t)} = \langle I_y \rangle^2 \int_\eta \overline{k^2(x,y + \eta,z,t)} \int_\epsilon R(\epsilon)\ d\epsilon d\eta \qquad (2.16)$$

where $R(\epsilon)$ is the space correlation coefficient. But,

$$\int_{-\infty}^{\infty} R(\epsilon)\ d\epsilon = L_y$$

the integral scale of turbulence in the radial direction. Hence

$$\overline{i_y^2(t)} = \langle I_y \rangle^2 L_y \int_\eta \overline{k^2(x,y + \eta,z,t)}\ d\eta \qquad (2.17)$$

Comparing this equation with Eq. (2.8a), we obtain

$$\frac{G(x,y,z)}{\overline{i_y^{\,2}(t)}} = \frac{\langle I_z \rangle}{\langle I_y \rangle} \; \frac{k^2(x,y,z)\; A(x,y,z)}{L_y \int_\eta k(x,y+\eta,z,t)\; d\eta} \tag{2.18}$$

Up to this point we have regarded the point (x,y,z) as the 'fixed' beam intersection in Fig. 1, for example, while η has been used to denote distance from this point. To complete the evaluation of the integral scale, however, we next need to consider Eq. (2.18) in terms of a series of intersection points extending over the complete flow cross section. The 'y' co-ordinate of the intersection point thus becomes a variable, and we write (2.18) as

$$\frac{\langle I_y \rangle}{\langle I_z \rangle} \; \frac{G(x,y,z)}{\overline{i_y^{\,2}(t)}} = \frac{A(x,y,z)\; \overline{k^2(x,y,z,t)}}{L_y \int_\eta k^2(x,y+\eta,z,t)\; d\eta} \tag{2.19}$$

and integrate both sides of this expression over the entire flow cross section to obtain

$$\int \frac{\langle I_y \rangle}{\langle I_z \rangle} \; \frac{G(x,y,z)}{\overline{i_y^{\,2}(t)}}\, dy = \frac{A(x,y,z) \int \overline{k^2(x,y,z,t)} dy}{L_y \int \overline{k^2(x,y+\eta,z,t)}\; d\eta} \tag{2.20}$$

It should be noted here that the quantities A(x,y,z) and L_y are written outside the integral sign following our previously introduced hypothesis that the turbulent scales are independent of radial location.

However, since the integrals on the rhs of Eq. (2.20) are functions only of their now identical limits and not of the variable of integration, Eq. (2.20) reduces to

$$\int \frac{\langle I_y \rangle}{\langle I_z \rangle} \; \frac{G(x,y,z)}{\overline{i_y^{\,2}(t)}}\, dy = \frac{A(x,y,z)}{L_y} \tag{2.21}$$

All quantities on the lhs of the equation are measurable parameters, while the rhs is the ratio of an integral correlation area to the integral scale of turbulence in the 'y' direction. It is therefore assumed that this ratio approximates the integral scale of turbulence in the remaining 'z' direction, i.e.,

$$L_z = \int \frac{\langle I_y \rangle}{\langle I_z \rangle} \frac{G(x,y,z)}{\overline{i_y^{\,2}(t)}} dy \qquad (2.22)$$

The definition of $A(x,y,z)$, Eq. (2.8b), shows this to be precisely true if $R(x,y+\eta,z+\zeta)$ is a separable function, while our experiments, under Contract NAS8-20408, have served to confirm that the use of the method represented by Eq. (2.20) does yield values of the integral scales which do show agreement with the expected values. Finally it is to be emphasized that this method for obtaining the radial integral turbulent scales does depend heavily on the assumption that these scales are independent of position in the plane containing the intersecting beams. It is therefore not applicable to situations in which one or both beams must traverse a number of dissimilar flow regimes.

To summarize, within the limits of the assumptions of the above analysis, we can obtain the required radial scale of the turbulence with the following procedure.

1. The beams are intersected at a chosen location in the flow and the four measurable quantities, namely the mean values of the beam intensities, the covariance of the fluctuations, and the mean square levels of the fluctuations at the 'fixed' 'y' beam are obtained to form the dimensionless integrand of Eq. (2.22).
2. This is repeated at a representative selection of points covering the entire flow cross section.
3. The resulting function is integrated, in practice, numerically to obtain the required scale L_z.

Obviously the remaining scale, L_y, can be subsequently obtained by keeping the vertical beam fixed and varying the horizontal one.

SUMMARY

The purpose of the foregoing sections has been to discuss the basic concepts involved in the optical crossed beam correlation method. It has been shown that a combination of absorption measurements with cross correlation analysis can be used to obtain estimates of local turbulent flow properties.

Equation (2.9) indicates that the measured covariance is directly proportional to the turbulent intensity at the beam intersection point. With the assumption that the integral length scales L_y and L_z are not strong functions of position for a given cross section of the flow, a relative turbulent intensity profile can be obtained directly by measuring this covariance for a series of beam intersection points. Obviously, the measured quantity is not strictly a point value, but represents a weighted average of the intensity over the locally correlated area. However, a calculation, in which empirical expressions were developed for the intensity profile and lateral space correlations as measured in a subsonic jet [7], shows that the weighting is such that the measured quantity follows the true intensity profile to an accuracy of 5 percent over the entire cross section of the flow.

To obtain the absolute level of the fluctuations of a flow property, the product $L_y L_z$ must necessarily be known, as well as the relationship between this property and the extinction coefficient. The latter can conveniently be obtained from static calibration measurements using a standard absorption cell, while a method for obtaining the radial scales is outlined above. The remaining kinematic flow properties, namely the integral scale in the flow direction, the turbulent spectrum, convection velocity, moving axes time scale, or eddy lifetime, can all be obtained from the space time correlation $r(\xi,\tau)$ which is given by Eq. (2.14) as the ratio of the measurable quantities.

$$r(\xi,\tau) = \frac{G(x+\xi,y,z,\tau)}{G(x,y,z)}$$

Thus, it appears in principle that many of the properties, which have been previously measured in subsonic flows using hot-wire anemometer techniques, can now be measured using the crossed beam correlation method. This method eliminates the necessity of inserting solid probes into the flow field thereby permitting measurements to be made over a wider range of turbulent flows than has been possible in the past.

It should also be pointed out that the possibility of choosing the radiation employed from any portion of the electromagnetic spectrum also offers a degree of flexibility not available with more standard methods. For example, in a multi-component or reacting flow suitable choice of the wavelength and wavelength interval of the radiation could be employed to monitor fluctuations of a particular component of interest.

CHAPTER 3

OPTICAL INTEGRATION OVER CORRELATION AREAS IN TURBULENT FLOWS

INTRODUCTION

The previous chapter of this report has demonstrated the way in which the various quantities measured, using a crossed beam correlation system, can be combined to yield estimates of pointwise turbulent properties. However, the discussion clearly indicates that the properties so measured are close estimates only since, in practice, the technique measures an integral over the correlation area surrounding the point of interest. Justification for interpreting the measurements as point properties is, of course, offered by the fact that normally the correlated area in question is small compared to the overall flow, and is reinforced by the fact that the integral is strongly weighted by contributions generated close to the beam intersection point. Nevertheless, the acceptance of these simplifying assumptions is necessary to such interpretation.

Commonly, however, when attempting to estimate the influence of a random field, the converse problem arises. Pointwise measurements of the properties of the field are available, but some type of space integral of these properties is required to specify the resulting effect. Let us proceed to demonstrate this feature in terms of an example chosen primarily for its well known nature.

THE LOAD ON A FLAT PLATE

Consider the problem of estimating the load on a flat plate due to a random pressure field. If the plate is located in the y,z plane and the pressure at any point can be represented by $p(y,z,t)$ then the load at time t is

$$L(t) = \int_z \int_y p(y,z,t)\, dy\, dz \tag{3.1}$$

However, since the load is a random function of time, some statistical representation of its magnitude and other characteristics is normally required. We shall consider its auto-correlation function $R(\tau)$ since the particular value $R(\tau=0)$ will yield the mean square amplitude, while Fourier transformation of the complete function will yield the spectral distribution. By definition

$$R(\tau) \equiv \overline{L(t)\, L(t+\tau)} \tag{3.2}$$

which, using (3.1), can be written

$$R(\tau) = \int_y \int_z \int_\eta \int_\zeta \overline{p(y,z,t) p(y+\eta, z+\zeta, t+\tau)}\, d\eta d\zeta dy dz \tag{3.3}$$

Let us finally consider the relatively simple situation of requiring only the contribution to this integral of the point (y,z) alone. This is

$$R(y,z,\tau) = \int_\eta \int_\zeta \overline{p(y,z,t)\; p(y+\eta, z+\zeta, t+\tau)}\, d\eta d\zeta \tag{3.4}$$

It is apparent therefore that even to estimate the contribution of the single point to the overall load a knowledge is needed of the cross correlation existing between it and all other points on the plate. To obtain the data needed for the exact evaluation of Eq. (3.4) represents a formidable experimental task and it is common to employ simplifying assumptions which reduce the amount of data required although it is still considerable. The situation becomes even more formidable, of course, if the required function is a volume rather than an area integral. It is the purpose of the subsequent two sections of this report to demonstrate the way in which a crossed beam system can be used to measure such functions with relative ease.

AREA INTEGRALS OF SPACE-TIME CORRELATION FUNCTIONS

Let us start by considering Eq. (2.10) rewritten for the case $\xi = 0$ in the form

$$\frac{G(x,y,z,\tau)}{\langle I_y \rangle \langle I_z \rangle} = \int_\eta \int_\zeta \overline{k(x,y+\eta,z,t)\; k(x,y,z+\zeta,t+\tau)}\; d\zeta d\eta \qquad (3.5)$$

There is obviously a strong functional similarity between this expression and Eq. (3.4). This functional similarity can be made exact if it is permissible to assume that the turbulent field is spatially homogeneous along one of the beam directions. For the sake of discussion, let this be the 'y' beam. Then,

$$\overline{k(x,y+\eta,z,t)\; k(x,y,z+\zeta,t+\tau)} = \overline{k(x,y,z,t)\; k(x,y-\eta,z+\zeta,t+\tau)} \quad (3.6)$$

and Eq. (3.5) becomes

$$\frac{G(x,y,z,\tau)}{\langle I_y \rangle \langle I_z \rangle} = \int_\eta \int_\zeta \overline{k(x,y,z,t)\; k(x,y+\eta,z+\zeta,t+\tau)}\; d\zeta d\eta \qquad (3.7)$$

The rhs of this expression obviously represents the integral of space time correlation function, existing for time delay τ, between the point (x,y,z) and all other points within the plane containing the beams. It is thus exactly the form of expression which our previous discussion of the loaded plate indicated was required.

Obviously the most questionable portion of our argument used to transform Eq. (3.5) into the required form of Eq. (3.7) is the assumption of flow homogeneiety along the 'y' beam direction. However, as discussed by Krause [14], this assumption is over restrictive and was used above only for the purpose of clarity.

Let us now consider the more general case in which the flow is inhomogeneous so that

$$\overline{k(x,y+\eta,z,t)\ k(x,y,z+\zeta,t+\tau)}$$

$$+\ \overline{k(x,y,z,t)\ k(x,y-\eta,z+\zeta,t+\ )} + h(\eta,\zeta,\tau) \qquad (3.7a)$$

where $h(\eta,\zeta,\tau)$ is therefore a measure for the inhomogeneity and is identically equal to zero for the homogeneous case. In substituting expression (3.7a) into (3.5), we find the error term, i.e., the difference between the value obtained for the homogeneous and inhomogeneous case respectively, is of the form

$$\int_{\zeta}\int_{\eta} h(\eta,\zeta,\tau)\ d\eta d\zeta \qquad (3.7b)$$

Invoking again arguments similar to those employed in the discussion of Eq. (2.8), it would again appear reasonable to assume that the variation of the inhomogeneity over the limited flow region for which the covariance is finite follows a linear trend. Since, by definition, $h(\eta,\zeta,\tau)$ is zero for $\eta = 0$ $h(\eta,\zeta,\tau)$ will be an odd function of η and hence the integral of expression (3.7b) is equal to zero.

Thus the transformation of Eq. (3.5) into the required form of Eq. (3.7) is still valid even in the inhomogeneous situation as long as the degree of inhomogeneity varies either linearly with translational distance from the beam intersection point or follows some other 'odd' functional relationship with respect to η. It is felt that since we need consider the variation only over the locally correlated area surrounding the beam intersection point deviations from this requirement will not be encountered frequently.

Furthermore, it is also important to realize that the accuracy with which area integrals of correlation functions can be determined with point probes is not high. Flow disturbance effects will normally preclude the use of a large number of probes simultaneously. Thus, many repeated runs, using pairs of probes, are normally required followed by considerable numerical integration of results in which errors tend to accumulate. Even then, assumptions of local homogeneity are often necessary to make the number of experiments manageable. By contrast, the required area integral is obtained from the crossed beam method in one experiment, which therefore offers a higher potential accuracy as well as considerable economic savings in the amount of data to be processed.

Let us next proceed to consideration of the even more formidable problem of obtaining a volume integral of the space time correlation function of the type required in Lighthill's [8] theory of aerodynamic noise generation. The form of the required expression might be written

$$F(x,y,z,\tau) \equiv \int_{\xi}\int_{\eta}\int_{\zeta} p(x,y,z,t)\ p(x+\xi,y+\eta,z+\zeta,t+\tau)\ d\zeta d\eta d\xi \quad (3.8)$$

Obviously, such an integral could be generated using the previously discussed form of the crossed beam system in which motion of one beam in the ξ direction is now employed. Equation (3.7) becomes

$$\frac{G(x+\xi,y,z,\tau)}{\langle I_y\rangle\langle I_z\rangle} = \int_{\eta}\int_{\zeta} \overline{k(x,y,z,t)\ k(x+\xi,y+\eta,z+\zeta,t+\tau)}\ d\zeta d\eta \quad (3.9)$$

and (3.8) can be generated by repeated experiments for the various ξ values with subsequent integration of the results. Although this measurement would require several experiments (i.e., one for each value of ξ), the number of measurements is at least one order of magnitude less than would be required using point probes.

However, we shall now proceed to demonstrate that, in principle, it is also possible to obtain the required volume integral in one experiment by the combination of a 'thick' and 'thin' beam of radiation.

VOLUME INTEGRALS OF SPACE-TIME CORRELATION FUNCTIONS

The experimental arrangement necessary for a 'one shot' estimate of a volume integral of the space-time correlation function about the point (x,y,z) is shown schematically in Fig. 3. A 'thin' beam of radiation similar to those considered previously is passed across the flow in the 'y' direction from source S_1 to detector D_1.

Thus, as shown previously (Eq. 2.4), the fluctuating signal at this detector at time t is

$$i_y(t) = \langle I_y \rangle \int_\eta k(x,y+\eta,z,t)\, d\eta \tag{3.10}$$

This beam intersects the second thick beam at the point of interest. This second beam, as shown in Fig. 3, is of appreciable length in the flow or x-direction emanating from the extended source S_2 and being monitored by the detector D_2.

Consider first the elemental column of this beam AB, which is at the position $(x+\xi)$. Referring again to the arguments used to obtain Eq. (2.4), the contribution to the total signal fluctuation at detector D_2 of this element at time $(t+\tau)$ is

$$i(\xi,t+\tau) = I(\xi) \int_\zeta k(x+\xi,y,z+\zeta,t+\tau)\, d\zeta \tag{3.11}$$

Obviously the total signal fluctuation at detector D_2 can be obtained by formal integration over ξ and is given by

$$i_z(t+\tau) = \int_\xi I(\xi) \int_\zeta k(x+\xi,y,z+\zeta,t+\tau)\, d\zeta d\xi \tag{3.12}$$

To obtain the required volume integral, it is necessary to impose a weak dependence of the quantity $I(\xi)$ on position along the thick beam. With this condition satisfied (3.12) becomes

$$i_z(t+\tau) = \langle I_z \rangle \int_\xi \int_\zeta k(x+\xi, y, z+\zeta, t+\tau)\, d\zeta d\xi \qquad (3.13)$$

where $\langle I_z \rangle$ is the mean level of the signal at detector D_2. If we now cross correlate the output from these two detectors, we obtain

$$G_I(x,y,z,\tau) =$$
$$\langle I_y \rangle \langle I_z \rangle \int_\xi \int_\eta \int_\zeta k(x, y+\eta, z, t)\; k(x+\xi, y, z+\zeta, t+\tau)\, d\zeta d\eta d\xi \qquad (3.14)$$

Finally, assuming the flow is homogeneous in the 'y' direction over a typical correlation length, we can use the transformation demonstrated in our previous discussion of area integration to obtain

$$G_I(x,y,z,\tau) =$$
$$\langle I_y \rangle \langle I_z \rangle \int_\xi \int_\eta \int_\zeta k(x,y,z,t)\; k(x+\xi, y+\eta, z+\zeta, t+\tau)\, d\zeta d\eta d\xi \qquad (3.15)$$

Thus the measurable ratio

$$\frac{G_I(x,y,z,\tau)}{\langle I_y \rangle \langle I_z \rangle}$$

is exactly the volume integral of space time correlation functions expressed in Eq. (3.8) and using the system shown in Fig. 3 it can be obtained from a single measurement.

It should be noted, however, that to obtain this result one additional condition over that required for an area integral is necessary; namely, that the mean intensity received at various positions along the extended detector must be constant. If this condition is not fulfilled, a certain amount of unwanted weighting of the volume integral by those portions of the flow where this intensity is high will result (see Eq. 3.12). The necessary length of the "broad beam" is also obviously an important feature in determining the extent to which this condition will be fulfilled. It must necessarily be of sufficient length that the turbulent pattern which originally passed through the point (x,y,z) is completely destroyed before it is convected out of this beam. In practice, this would involve a beam length of order five shear layer widths. Our experiments on supersonic free shear layer (Contract NAS8-20408) do indicate that in these flows the change of mean absorption with streamwise position would be acceptable. In flows where appreciable changes of mean absorption with position do occur, another solution, in principle, is to use a weak absorption process. Here the mean intensity is kept appreciably constant in spite of absorption changes.

Finally, it must be emphasized that no attempts to use this proposed method for the measurement of volume integrals of space time correlation functions has been made to date. Thus, the practical problems of generating the necessary uniform broad beam or of measuring the required correlations of the two detected signals have not been considered in any detail.

A further effect which requires consideration here is the influence of scattering of the radiation by particulate matter in the flow. While this is allowed for in the extinction coefficient definition for two thin beams, the possibility of multiple scattering and its influence on the measurement in the case of the thick beam proposed here does require further analysis. The possibility of using a proposed infrared crossed beam system, in which the longer wavelength would offer some reduction of the influence of scattering, offers one possible solution to this latter problem.

MEASUREMENT OF THE THREE DIMENSIONAL WAVE NUMBER SPECTRUM

The previous two sections of this report have demonstrated the way in which the local integrating features of a crossed beam system can be employed to generate, very efficiently, two types of functions which are of great practical importance in the applications of turbulence work. We conclude this chapter with a discussion of the way in which the technique may also be employed to obtain a very fundamental quantity in the statistical theory of turbulence; namely, the three dimensional wave number spectrum. This quantity, as we shall demonstrate below, is very important in defining the spatial structure of a turbulent flow. However, its measurement is not possible with point probes, except indirectly in isotropic turbulence where the three dimensional function can be calculated from the one dimensional spectrum function which is measured by the point probe. However, no unique relationship, between the measurable one dimensional function and the required three dimensional function, exists for the more general class of anisotropic turbulent flows. This difficulty has led to many problems in the interpretation of turbulence data in terms of the spatial sizes of the disturbances which contribute to the overall turbulent energy.

The problem can be simply characterized as follows. Consider two probes in a turbulent flow as shown in Fig. 4a with which we desire to measure the streamwise component of the three dimensional spectrum function $E(\kappa_x,0,0)$. That is, we choose to regard the flow as comprising of plane waves travelling in the streamwise direction and we wish to investigate the way in which the total energy of the turbulence is distributed among the various wavelengths. For discussion purposes, we will imagine that these two probes are capable of selecting energy only from disturbances whose wavelengths correspond to their separation, an example of which is shown in Fig. 4a. Thus, by using various separations and measuring the energy per unit time recorded for each separation, the required wave number spectrum could be obtained. However, consider next the situation shown in Fig. 4b in which the passage of a cross flow component is registered by the probes. The probes, having no knowledge that this disturbance is not travelling in the streamwise direction, attribute this energy contribution to a wavelength λ_i, where, as shown, the wavelength is in fact λ_a. It is therefore apparent that cross flow components of wavelength λ_a will always contribute energy to the one dimensional spectrum function at a wavelength $\lambda > \lambda_a$.

For the case of isotropic turbulence a unique relationship between the one and three dimensional system function does exist and is given by [9] as

$$E(\kappa_x,0,0) = \frac{1}{4}\int_{\kappa_x}^{\infty} \frac{1}{\kappa^3} E(\kappa)\ (\kappa^2 - \kappa_x^2)\ d\kappa \qquad (3.16)$$

It is perhaps interesting to consider, in terms of a simple example, the way in which interpretation of a wave number spectrum becomes confused if only the one dimensional function is measurable. We consider, in order to use expression (3.16) above, a hypothetical isotropic turbulent flow in which only one wave number component is present. Measurement of the three dimensional spectrum function would then clearly yield the rather simple wave number spectrum shown in Fig. 5a. However, using expression (3.16), it is easily shown that the corresponding one dimensional spectrum function would take the form shown in Fig. 5b. Thus blind interpretation of this measurement would suggest the equi-partition of the turbulent energy in all wave numbers from the value actually present down to zero. In an isotropic flow the problem can be resolved, but in practice we are far more interested in anisotropic flows, where no unique relationship like (3.16) exists. Thus having measured the one dimensional spectrum function, doubt must always remain as to the extent to which the energy at low wave numbers is contributed by large scale disturbances travelling in the streamwise direction and the extent to which it is contributed by relatively small cross flow components. It is the opinion of this author and demonstrated by the example cited above that the rather complicated picture of the structure of turbulent shear flows which currently exists may well become considerably simplified when measurements of the three dimensional spectrum function become available.

Let us now consider the formal definition of this function and proceed to demonstrate the way in which it can be measured using a crossed-beam system.

Following Hinze [10] the function is defined

$$E(\kappa_x,\kappa_y,\kappa_z) = \frac{1}{8\pi^3}\iiint_{-\infty}^{\infty} \overline{v(x,y,z,t)\; v(x+\xi,y+\eta,z+\zeta,t)}$$

$$\exp\,[-i(\kappa_x\xi+\kappa_y\eta+\kappa_z\zeta]\; d\zeta d\eta d\xi \tag{3.17}$$

Considering therefore the distribution of energy among the streamwise, κ_x, wave numbers this may be written

$$E(\kappa_x,0,0) = \frac{1}{8\pi^3}\iiint_{-\infty}^{\infty} \overline{v(x,y,z,t)\; v(x+\xi,y+\eta,z+\zeta,t)}\; e^{-i\kappa_x\xi}\, d\zeta d\eta d\xi \tag{3.18}$$

Comparing the rhs of this expression with (3.9), we find

$$E(\kappa_x,0,0) = \frac{1}{8\pi^3}\int_{\xi} \frac{G(x+\xi,y,z,\tau=0)}{\langle I_y\rangle\langle I_z\rangle}\, e^{-i\kappa_x\xi}\; d\xi \tag{3.19}$$

Thus referring to Fig. 2, the required spectrum function can be obtained by measuring the covariance between the two detected signals as a function of the streamwise beam separation ξ. Standard Fourier transform routines can subsequently be applied to this function to obtain the required spectrum. Clearly the evaluation of the single integral involved in (3.19) is far more practical than evaluation of the triple integral of (3.18) which would be necessary using local probes and which has precluded measurement of the three dimensional spectrum function to date.

SUMMARY

Our discussion of Chapter 2 showed that the crossed correlation of optical signals can be employed to obtain useful estimates of local turbulent properties.

Although there is a strong tendency to attempt to characterize a turbulent field in terms of such local or pointwise properties, this is not always necessary or desirable. When we wish to evaluate the effectiveness of a turbulent region as a forcing function, a considerable amount of integration of pointwise properties over a correlated region is normally necessary. The bulk of information necessary to make such an evaluation from pointwise determined quantities is very considerable, and certain simplifying assumptions are normally introduced to reduce it to a manageable amount. The crossed beam correlation method on the other hand performs a considerable amount of the required integration automatically and the measured quantities often resemble very closely the required integrals. Thus, the evaluation of the effects of regions of turbulence can be performed far more efficiently than would be the case if only point probe information were available.

Finally, the possibility to measure the three dimensional spectrum function, as opposed to the one dimensional function available from point probe measurements, will prove a useful feature in furthering our basic understanding of anisotropic turbulent shear flows.

CHAPTER 4

PRACTICAL CONSIDERATIONS OF THE CROSSED-BEAM METHOD

INTRODUCTION

In the previous two chapters of this report the theoretical concept and potential capabilities of the crossed-beam method have been introduced. To avoid confusion these concepts have been presented for a rather idealized situation. For example, it has been tacitly assumed that the fluctuations of light intensity generated by the flow are detectable without consideration being given to the conditions for which this is true. No consideration has been given to the effects of either light source fluctuations or detector noise, while finally it has been assumed that the required covariance between the detector signals can be measured irrespective of its magnitude relative to, for example, the magnitude of the independent detector signals. Consideration of these more practical aspects of utilizing the method are the prime consideration of this chapter.

OPTIMIZATION OF THE EXTINCTION COEFFICIENT

Consider the beam S_1D_1 passing across the flow as shown in Fig. 1. It has been assumed that the radiation is such that it is either absorbed or scattered by a flow constituent, while fluctuations of the flow properties lead to fluctuations of the extinction coefficient and hence detected light intensity. Clearly two extreme and undesirable situations could exist. First, if the extinction process is very weak the resultant signal at the detector will be comprised of a large mean level on which small fluctuations are superimposed. Alternatively if the extinction process is extremely powerful, both the mean and fluctuating signal at the detector will be extremely small. One hopes that between these two extremes an optimum situation exists and the subsequent analysis, in fact, indicates that this is the case.

Consider Eq. (2.2) and let us define a number n(t) which is the ratio of the fluctuation of total extinction along the path to the time averaged value, i.e.,

$$n(t) = \frac{\int k(x,y+\eta,z,t)d\eta}{\int \langle K(x,y+\eta,z)\rangle d\eta} \tag{4.1}$$

We can then rewrite (2.2) in the form

$$I_y(t) = I_o e^{-\int \langle K(x,y+\eta,z)\rangle d\eta} e^{-n(t)\int \langle K(x,y+\eta,z)\rangle d\eta} \tag{4.2}$$

Temporal integration of this equation will show that to a first degree of approximation

$$\langle I_y \rangle \simeq I_o e^{-\int \langle K(x,y+\eta,z)\rangle d\eta} \tag{4.3}$$

Thus Eq. (4.2) can be written

$$I_y(t) = \langle I_y \rangle \left\{\frac{\langle I_y \rangle}{I_o}\right\}^{n(t)} \tag{4.4}$$

But by definition

$$I_y(t) = \langle I_y \rangle + i_y(t) \tag{4.5}$$

Thus the fluctuating signal at the detector $i_y(t)$ is

$$i_y(t) = \langle I_y \rangle \left\{\frac{\langle I_y \rangle}{I_o}\right\}^{n(t)} - \langle I_y \rangle \tag{4.6}$$

We wish to determine therefore what value of $\langle I_y \rangle$, if any, will make the magnitude of $i_y(t)$ a maximum for a given level of fluctuation (i.e., value of n(t)).

Differentiating (4.6) with respect to $\langle I_y \rangle$ yields

$$\frac{\partial i_y(t)}{\partial \langle I_y \rangle} = \frac{[n(t) + 1] \langle I_y \rangle^{n(t)}}{I_o^{\,n(t)}} - 1 \tag{4.7}$$

Thus the condition which must be satisfied to yield the largest fluctuation in detected signal level for a given relative fluctuation of extinction coefficient, n(t), is

$$\frac{\langle I_y \rangle}{I_o} = \left\{\frac{1}{n(t) + 1}\right\}^{1/n(t)} \tag{4.8}$$

It is apparent from this equation that the optimum degree of mean absorption, $\langle I_y \rangle / I_o$, is dependent on the amplitude of the fluctuations. Fortunately, however, the dependence is not strong at least for fluctuations of order 50 percent or less of the mean value as is shown by Table 1 below.

TABLE 1

Value of n(t)	$\frac{\langle I_y \rangle}{I_o}$ Optimum
0.01	0.370
0.05	0.377
0.1	0.385
0.25	0.409
0.5	0.445
0.75	0.475
1	0.500

It is of interest to notice here that as the amplitude of the fluctuations increases it is desirable to decrease the amount of mean extinction. However, it appears that in the range in which one would normally be interested ($0.01 < n(t) < 0.25$) a value of $\langle I_y \rangle / I_o$ of order 0.37 would be most suitable. This, following Eq. (4.3), requires

$$\int \langle K(x,y+\eta,z) \rangle \, d\eta \simeq 1 \tag{4.9}$$

It is to be emphasized that the ability to satisfy Eq. (4.9) is not a prerequisite to the application of the crossed-beam method. However, if small fluctuations are anticipated and an experimentally convenient control of the extinction process is available, it does provide a guide to the amount of mean light attenuation for which one should aim.

A choice of the degree of mean extinction is most liable to arise in two ways. First, if absorption is to be used, the absorbing species might exhibit a wide range of absorption coefficient as a function of optical wavelength of the radiation. Obviously Eq. (4.9) should be satisfied if in so doing no other penalties are involved. However, in practice this could involve working in a wavelength region where light source brightness is inadequate or it might involve using an extremely narrow wavelength interval, which would also, in general, involve penalties of beam intensity. Alternatively use of another wavelength or wavelength interval might alleviate these problems at the expense of contravening to some extent Eq. (4.9). In such circumstances no general rules for choosing the degree of mean absorption can be layed down and each experimental situation must be considered on its own merits. A second situation in which control of the degree of beam attenuation is available arises when scattering from particulate matter, artificially introduced into the flow, is employed. Here introduction of sufficient tracer can always be used to ensure that the optimum amount of mean attenuation is obtained. On the other hand, introduction of 'sufficient' tracer might be experimentally inconvenient or expensive while the higher degree of flow contamination might influence the flow properties. Once more, therefore, each situation must be considered on its own merit.

In conclusion, experience has shown that in the experiment design stage Eq. (4.9) is extremely valuable in defining an optimum situation. However, a wide range of compromise around the optimum is tolerable and is often experimentally convenient. To demonstrate this point we present Table 2 in which the magnitude of the fluctuating signal as a function of the degree of mean extinction is presented for the particular case n(t) = 0.1 (see Eq. 4.6).

TABLE 2

$\frac{\langle I_y \rangle}{I_o}$	$\frac{i_y(t)}{I_o}$	$\frac{i_y(t)}{\langle I_y \rangle}$	$\int \langle K(x,y+\eta,z) \rangle d\eta$
0.01	0.0037	0.370	4.6
0.05	0.013	0.260	3.0
0.10	0.0206	0.206	2.3
0.20	0.030	0.150	1.61
0.30	0.034	0.11	1.17
0.40	0.036	0.09	0.92
0.50	0.034	0.068	0.69
0.60	0.030	0.050	0.51
0.70	0.025	0.0357	0.357
0.80	0.018	0.0225	0.222
0.90	0.0095	0.010	0.106

This table clearly indicates the wide range of extinction coefficients which are tolerable and can be used should they prove experimentally more convenient than the optimum.

EFFECTS OF LIGHT SOURCE FLUCTUATIONS AND DETECTOR NOISE

In Chapter 2 of this report, relationships have been established between local fluctuations of flow properties and the covariance of the fluctuations at two independent photodetectors. However, for the sake of brevity and clarity, both the light sources and detectors have been considered as noise free. In any practical system, this will not be the case and it is the purpose of this section to consider the effects of both light source fluctuations and detector noise on the required measurements.

Let us first rewrite Eq. (2.1) in a form which allows for both temporal fluctuations of source intensity and for the presence of noise in the detector, i.e.,

$$I_y(t) = I_o(t)\, e^{-\int K(x,y+\eta,z,t)d\eta} + v_{ny}(t) \tag{4.10}$$

where $v_{ny}(t)$ is the noise amplitude in the detector at time t.

Rewriting the source intensity in terms of its time averaged value I_o and a fluctuation relative to the mean $i_o(t)$ Eq. (4.10) becomes

$$I_y(t) = I_o e^{-\int K(x,y+\eta,z,t)d\eta} + i_o(t) e^{-\int K(x,y+\eta,z,t)d\eta} + v_{ny}(t) \tag{4.11}$$

Following essentially similar arguments to those employed to obtain Eq. (2.4) from Eq. (2.1), a little algebraic manipulation is sufficient to show that the instantaneous fluctuation at the detector is given by

$$i_y(t) = \langle I_y \rangle \int k(x,y+\eta,z,t)d\eta + \frac{\langle I_y \rangle}{I_o} i_o(t)$$

$$+ \frac{\langle I_y \rangle}{I_o} i_o(t) \int k(x,y+\eta,z,t)\, d\eta + v_{ny}(t) \qquad (4.12)$$

It is interesting to identify the source of these four terms. The first, clearly (see Eq. 2.4a), is the required modulation of the light beam created by the turbulent flow. The second represents the fluctuations of source intensity after they have been attenuated by the mean flow. The third represents the modulation of these fluctuations by the turbulent flow. However, a brief comparison of this with the first term indicates it will be small as long as the light source fluctuations are not a large percentage of the mean value. Since this will generally be the case, it will not be considered further. Finally, by definition, $v_{ny}(t)$, is the noise signal at the detector.

Thus we can write the resultant signal at the detector as the sum of three contributions

$$i_y(t) = v_{fy}(t) + v_{oy}(t) + v_{ny}(t) \qquad (4.13)$$

Similarly, for the second detector we obtain

$$i_z(t) = v_{fz}(t) + v_{oz}(t) + v_{nz}(t) \qquad (4.14)$$

where $v_{fz}(t)$ represents the required fluctuations created by the flow, $v_{oz}(t)$ is the result of the second light source fluctuations attenuated by the flow, and $v_{nz}(t)$ is the noise signal from this second detector.

The covariance of the two detected signals, $G(x,y,z)$, is then given in terms of a total of nine time averaged products comprising:

(a) $\overline{v_{fy}(t)v_{fz}(t)} = \langle I_y \rangle \langle I_z \rangle \int_\eta \int_\zeta \overline{k(x,y+\eta,z,t)k(x,y,z+\zeta,t)} d\eta d\zeta$

which is the required quantity given in Eq. (2.6).

(b) $\overline{v_{oy}(t)v_{oz}(t)}$ which represents the covariance between the fluctuations at the two sources. This will be zero if these fluctuations are mutually random.

(c) $\overline{v_{ny}(t)v_{nz}(t)}$ similarly represents the covariance between the noise signals generated by the two detectors and will also have a value zero if no correlation exists between them.

The subsequent four terms clearly represent the degree of coherence existing between fluctuations generated by the flow in one beam and the light source and detector noise fluctuations of the other system. That such a correlation should exist seems very unlikely so that these terms similarly fail to contribute to the value of G(x,y,z).

Finally, the remaining two terms represent the correlation between light source fluctuations on one beam and the noise signal generated by the detector which monitors the other beam. Although it is not inconceivable that these fluctuations are interrelated, the probability is low and hence these terms similarly leave the value of G(x,y,z) unaffected.

Thus, it becomes clear that the existence of additional fluctuations (noise) in a crossed beam system, created either by fluctuations of light source intensity or within the detectors do not influence the value of the required covariance as long as they are not correlated.

The most likely source of unwanted <u>correlated</u> fluctuations would appear to be sources (b) and (c) above and their elimination leads to the following general recommendations:

1. The two light sources required for a crossed-beam experiment should be operated from separate power sources thus eliminating the possibility that fluctuation of a common power source could introduce correlated fluctuations in the two lamp outputs.
2. Similarly, any power sources required for operation of the photo-detectors should be independent of each other.

In practice in any laboratory experiment a potential source of a common fluctuation is that provided by the electrical mains. It is of the utmost importance in any crossed-beam experiment to eliminate these effects either at source or by subsequently removing this frequency from the recorded signal when this is permissible.

However, once these precautions are maintained it is an interesting and valuable feature of the crossed beam method that erroneous signals at the two independent detectors in no way influence the final result; namely, the covariance of these signals.

In practice, of course, there is an upper limit on the degree of erroneous noise which can be tolerated. However, as we shall now proceed to demonstrate, this influences primarily the statistical accuracy of the results as opposed to leading to erroneous conclusions. Further, signal to noise ratios several orders of magnitude lower than those which would be tolerable in more conventional methods are acceptable.

REQUIRED ACCURACY OF CORRELATION MEASUREMENTS

From previous discussion, it has become clear that the basic measurement to be performed in any crossed-beam system is the determination of the cross-correlation function of the two detector outputs, i.e.,

$$\frac{1}{T}\int_0^T i_y(t)\, i_z(t+\tau)\, dt$$

Further, the discussion of the previous section has shown that the value of this covariance is not influenced by either source or detector noise as long as these additional fluctuations are mutually random. Many practical situations will exist for which this condition is fulfilled, shot noise limited detectors for example. Under these conditions crossed-beam measurements are, in principle, possible irrespective of the magnitude of the noise. In practice, of course, there is a limitation which arises due to the necessity of using a finite integration time, T, over which the required correlation is estimated. The effect of this finite integration period is to leave some uncertainty in the value of the correlation function. Thus measurements are possible only if the acceptable integration period is of sufficient length to reduce this uncertainty to an acceptably small fraction of the value of the function itself.

To review this problem in a quantitative fashion, it will be convenient to define the correlation coefficient of the two detector signals

$$R(\tau) = \frac{\overline{i_y(t)\ i_z(t+\tau)}}{\left\{\overline{i_y^{\,2}(t)}\ \overline{i_z^{\,2}(t)}\right\}^{1/2}} \tag{4.15}$$

while Bendat [10] has shown that for a period of integration, T, an uncertainty will exist in the value ot $R(\tau)$ given by

$$\sigma = \frac{c}{\sqrt{bT}} \qquad \text{where} \qquad 1 < c < \sqrt{2} \tag{4.16}$$

where b is the (radian) bandwidth of the signals.

Bendat's relation (4.16) was, in fact, calculated for a particular spectral distribution of the signal energy; namely,

$$W(\omega) = \frac{b}{\omega^2 + b^2} \tag{4.17}$$

with a corresponding auto-correlation coefficient of the form

$$R(\tau) = e^{-b|\tau|} \tag{4.18}$$

However, work on the present contract [10] has also considered a band limited white noise spectrum of the form

$$\begin{aligned} W(\omega) &= 0 \qquad 0 < \omega < \omega_1 \\ W(\omega) &= 1 \qquad \omega_1 < \omega < \omega_2 \\ W(\omega) &= 0 \qquad \omega_2 < \omega < \infty \end{aligned}$$

It is shown that for this case an uncertainty of the form

$$\sigma = \frac{\sqrt{2\pi}}{\sqrt{(\omega_2 - \omega_1)T}} \tag{4.19}$$

results, which is very similar to (4.16).

Let us now consider an artificial type of crossed-beam experiment shown schematically in Fig. 6. Each cell is imagined to contain an absorbing species, while fluctuations of the degree of absorption can also be introduced. These fluctuations are mutually random in each cell, but are entirely coherent across a particular cell. Following Eq. (2.4) the fluctuating signal at detector 1 can be written

$$i_1(t) = v_{m1}(t) + v_{m2}(t) + \ldots . \; v_{mn}(t) + \ldots . \; v_{mN}(t) \tag{4.20}$$

where, for example,

$$v_{m1} = \langle I_1 \rangle k_{m1}(t) \ell$$

where $\langle I_1 \rangle$ is the mean intensity at detector 1, $k_{m1}(t)$ is the fluctuation of extinction coefficient in cell (m,1) at time t and ℓ is the width of the cell.

Similarly, for the second beam we can write

$$i_2(t) = v_{1n}(t) + v_{2n}(t) + \dots \; v_{mn}(t) + \dots \; v_{Nn}(t) \qquad (4.21)$$

The correlation coefficient for zero time delay is, from Eq. (4.15),

$$R(0) = \frac{\overline{i_1(t)\; i_2(t)}}{\left\{\overline{i_1^{\,2}(t)}\;\overline{i_2^{\,2}(t)}\right\}^{1/2}} \qquad (4.22)$$

Since we have assumed that the fluctuations in the independent cells are uncorrelated

$$\overline{i_1(t)\; i_2(t)} = \overline{v_{mn}(t)\; v_{nm}(t)} = \overline{v_{nm}^{\;2}(t)} \qquad (4.23)$$

$$\overline{i_1^{\,2}(t)} = \overline{v_{m1}^{\;2}(t)} + \overline{v_{m2}^{\;2}(t)} + \dots + \overline{v_{mN}^{\;2}(t)} \qquad (4.24)$$

$$\overline{i_2^{\,2}(t)} = \overline{v_{1n}^{\;2}(t)} + \overline{v_{2n}^{\;2}(t)} + \dots + \overline{v_{Nn}^{\;2}(t)} \qquad (4.25)$$

If now, purely for reasons of simplicity, we assume that the mean square level of the fluctuations in each cell are identical and equal to $\overline{v^2(t)}$ (4.22) becomes

$$R_i(0) = \frac{\overline{v^2(t)}}{N\overline{v^2(t)}} = \frac{1}{N} \qquad (4.26)$$

Thus the correlation coefficient to be measured is the reciprocal of the number of cells through which a single beam passes.

It is perhaps of interest at this stage to pause to draw the analogy between this hypothetical experiment and the more practical situation considered in Chapter 2. By definition, each cell is a region over which the fluctuations are coherent and thus is similar to the previous concept of a correlation area. Thus the length of a cell is the analogy of the integral length scale. In fact, consideration of Eq. (2.19) when the level of the extinction coefficient fluctuations are assumed independent of position along the beam will indicate that the correlation coefficient

$$\frac{G(x,y,z)}{\overline{i_y^{\,2}(t)}}$$

is the ratio of the integral scale to flow width or the number of scales contained in the beam. Further, it is apparent from the considerations above that the covariance of the signals in our hypothetical experiment is

$$\overline{i_1(t)\; i_2(t)} = \langle I_1\rangle\langle I_2\rangle\; \overline{k_{mn}^{\,2}}\; \ell^2 \tag{4.27}$$

which is the counterpart of Eq. (2.9).

Of course in obtaining relationship (4.26) the simplifying assumption that the intensity of the fluctuations is independent of position along the beam was introduced. In practice, there will normally exist a variation of this parameter. Consideration of Eqs. (4.22), (4.23), and (4.24) in conjunction with Eq. (4.26) indicates that in regions of high intensity the correlation coefficient will be higher than given by (4.26) while in regions of low intensity the converse will be true. However, Eq. (4.26) in which the value of N is the ratio of flow width to the integral scale will often yield a useful estimate of the typical magnitude of the correlation coefficients to be measured.

Finally, it is now a simple matter to consider the effect of additional uncorrelated noise on the magnitude of the correlation coefficient. We merely add the appropriate signal power generated by this noise, $\overline{v_n^{\,2}(t)}$, to the rhs of expressions (4.24) and (4.25). Eq. (4.26) then becomes

$$R_m(0) = \frac{\overline{v^2(t)}}{\overline{Nv^2(t)} + \overline{v_n^2(t)}} \tag{4.28}$$

which can alternatively be written

$$R_m(0) = \left(1/N\right) \cdot \frac{1}{\left\{1 + \frac{\overline{v_n^2(t)}}{\overline{Nv^2(t)}}\right\}} \tag{4.29}$$

It should be noted here that $\overline{Nv^2(t)}$ is the signal power generated at the detector by the required fluctuations of flow properties, while $\overline{v_n^2(t)}$ is the noise power. Thus the ratio of the correlation coefficient which would be obtained in an ideal noise free system, $R_i(0)$, to that which would be measured in a system containing noise, $R_m(0)$, would be

$$\frac{R_i(0)}{R_m(0)} = 1 + \frac{n}{s} \tag{4.30}$$

where n/s is the noise to signal ratio. Thus a signal to noise ratio of unity would have no more serious effect than reducing the magnitude of the correlation coefficient by a factor of two, which would, in normal circumstances make the experiment marginally more difficult. This is in vivid contrast to the more normal circumstance in which a signal to noise ratio of order 10^4 would be required if the facility to eliminate noise by correlation were not available.

EFFECTS OF FLOW STRUCTURE AND NOISE ON INTEGRATION TIME

We have seen above that the principal factor affecting the typical magnitude of the correlation coefficient is the number of integral turbulent scales through which the beams must pass. Erroneous noise generated either by light source fluctuations or detector noise will also reduce its magnitude, but the effect does not become significant until the noise level approaches that of the signal.

It is extremely difficult to establish those periods of integration which are needed to perform a crossed-beam experiment in terms of a general class of experiments. This difficulty arises primarily due to the fact that either a change in turbulent scale or in flow velocity alters the parameter 'b' in Eq. (4.16), resulting in a change of the integration time, T, necessary to yield a certain statistical accuracy in the covariance or correlation coefficient.

Alternatively, general criteria can be developed if we are prepared to work in a spatial, as opposed to temporal, frame of reference. If the signal characterized by the correlation function

$$R(\tau) = e^{-b|\tau|} \tag{4.18}$$

is generated by the passage of a convected turbulent pattern through the beams, then the time delay, τ, is equivalent to a beam separation x where

$$\tau = \frac{x}{U_c} \tag{4.31}$$

Thus the space correlation coefficient is

$$R(x) = e^{-bx/U_c} \tag{4.32}$$

and the integral scale is

$$L_X = \int_0^\infty R(x)\, dx = \frac{U_c}{b} \tag{4.33}$$

Further we can define a length, X, which is the product of the convection speed, U_c, and the integration time T. It thus represents the 'length' of flow which passed through the measuring point in time T.

Considering now Eq. (4.16), i.e.,

$$\sigma = \frac{c}{\sqrt{bT}}$$

we can substitute

$$b = \frac{U_c}{L_x} \tag{4.34}$$

and

$$T = \frac{X}{U_c} \tag{4.35}$$

so that the uncertainty becomes

$$\sigma = c\sqrt{\frac{L_x}{X}} \tag{4.36}$$

Thus we obtain the result that the uncertainty in a covariance estimate is proportional to the square root of the ratio of the integral scale to the 'length' of the flow which passed through the measuring point. It is felt that this result is considerably more general than its derivation might suggest. It states that the uncertainty of the covariance estimate is inversely proportional to the square root of the number of statistically independent events which were measured. This is a well known result for statistical work in which ensemble, as opposed to temporal, averages are involved.

Finally it is of interest to combine the results of Eq. (4.29) with that of Eq. (4.36) above to investigate the fractional certainty of the covariance estimate (i.e., the functional dependence of σ/R). We find

$$\frac{\sigma}{R_m(0)} = c\sqrt{\frac{L_x}{X}}\; N\left(1 + \frac{n}{s}\right) \qquad (4.37)$$

Since, for high accuracy of measurement, we require $\sigma/R_m(0)$ to be small, we can summarize the considerations of the latter portion of this chapter as follows: To obtain a specified percentage accuracy of our covariance estimates, it will be necessary to increase the integration length (and hence integration time for a given flow speed) if

(a) The integral scale in the flow direction is large.

(b) The number of integral scales traversed by the beam is large.

(c) The power at the detector, generated by a combination of light source fluctuations and detector noise, is of order or larger than the power contained in those fluctuations which are attributable to the flow.

Of these three considerations, both our experimental experience and Eq. (4.37) suggest that it is criterion (b) which is of primary concern. The existence of many independent fluctuations along the length of the beams will result in the necessity of measuring small correlation coefficients thus requiring a high statistical certainty and hence long integration periods.

STATUS OF CORRELATION MEASURING EQUIPMENT

Although Eq. (4.37) indicates that the availability of a sufficiently large integration time (i.e., value of X) will permit any required value of the ratio $\sigma/R_m(0)$ to be obtained limitations of available measuring instruments are not reflected in this equation.

In terms of analog equipment, the present state-of-the-art permits reliable measurements of correlation coefficients down to values of order 0.03. Beyond this our present experience with a Honeywell 9410 time delay correlator indicates that measurements are made extremely difficult and tedious due to drift of the reference levels of the instrument output. Increase of integration time required to theoretically improve the accuracy of the covariance measurement cannot be used here and thus the basic limitation to measurements in excess of a few percent appear inherent in such analog equipment.

On the other hand, improvement in the state-of-the-art of analog correlation equipment are currently being made due to a widespread and renewed interest in this type of measurement. It is the opinion of this author that at least an order of magnitude improvement can be expected, probably with the use of special purpose hybrid systems. It would appear that the basic multiplications required can probably be performed most efficiently and with acceptable accuracy using analog systems. However, the more critical integration procedures requiring long term stability of system components are probably better performed by more reliable digital systems.

An alternative solution, and the one which was adopted at the outset of this program, is to convert the information to digital form immediately following the acquisition phase. It is then possible to utilize the high degree of accuracy and reliability available on present day digital computers throughout the necessary data processing procedures.

At the outset of this work the Random Vibration Analysis program, RAVAN, previously developed by the Marshall Space Flight Center was utilized. Although a considerable volume of data was generated using this program, it also became apparent that the rather short integration periods available with this program would offer severe limitations in further extensions of the work. This limitation arose basically due to the necessity of storing within the computer all data to be processed. In view of the size of the basic program itself, this was restricted to 2,000 data samples.

This limitation has since been overcome with the development of the "Piecewise Correlation Program." This program has been developed by personnel of the George C. Marshall Space Flight Center with consulting services offered by personnel of the IIT Research Institute as called for by this contract.

The basic mode of operation of this program is as follows:

(a) A relatively short sample of the data to be processed is read into the computer and the required correlation function is calculated.
(b) A further sample of data is introduced and a new estimate of the function is obtained.
(c) These two estimates are then averaged and this updated average is compared with the first estimate.
(d) If the updated and previous estimate differ by more than a certain specified amount, a further sample is introduced and a further updating of the estimate is undertaken.
(e) This process is continued until the standard deviation of successive estimates is reduced to a certain specified value.

This program has the following specific advantages over the previous program:

(a) The limitation on the length of record which can be processed is completely removed.
(b) Not only is the final estimate of the correlation function obtained, but in addition an

estimate of the statistical certainty and hence significance of the answer is obtained.

(c) Frequent, automatic checking of the statistical accuracy of the correlation function means that no more computer time is utilized than that necessary to obtain the accuracy requested by the user.

This latter feature is extremely valuable in view of the relatively high cost of computer time.

At this time it is exceedingly difficult to estimate the ultimate capabilities of this program. The large amount of experimental data which has been processed since its inception in November 1966 has not permitted the amount of controlled testing for which one might wish. On the other hand, the results obtained from the reduction of some two hundred runs of data generated under Contract NAS8-20408 were of excellent quality.

It appears at this time that there are two factors which set the lower limit of correlation coefficients which may be measured using this digital analysis method. The first is the quantization error involved in the analog to digital conversion routine. Currently, seven bit digital conversion is employed meaning that the signal is assigned to one of one hundred and twenty-eight discrete levels. In the event that the correlated portion of the signal is buried within the total signal at a level less than the quantization step, its presence may go undetected and no correlation will result. A second feature, which results from the necessity of using long integration times to obtain good statistical accuracy for small correlation coefficients, is the basic stationarity of the data itself. These non-stationarities may result either from drifts in the crossed-beam apparatus itself (i.e., show changes of light source intensity for example) or due to non-stationarities of the measured process. These latter problems are most likely to arise in applications where relatively short duration running is necessitated by air storage limitations. The degree of limitation imposed by these considerations, however, still remains to be established and no quantitative values can be assigned at this time.

SUMMARY

The primary purpose of this Chapter has been to extend the rather idealistic treatment of the crossed-beam concept presented in Chapters 2 and 3 to a consideration of the more practical requirements. These can be summarized as follows:

(a) In choosing the extinction process which creates the required fluctuations of detected light intensity, a reduction of beam intensity of 60 percent between source and detector offers an optimum. However, a wide range around the optimum is available.

(b) The presence of light source fluctuations or detector noise in a crossed-beam system does not effect the value of the required correlation function as long as these fluctuations are mutually random between the two beams. The practical effects of system noise is to reduce the value of the correlation coefficient, thus requiring higher accuracy of the correlation system. In sharp contrast to many other methods, however, system noise is of serious practical concern only when its power exceeds that of the genuine signal.

(c) Perhaps the parameter of primary concern in establishing the feasibility of a given crossed-beam application is the number of integral scales of turbulence through which the radiation must pass between the source and detector. As shown in Eq. (4.26), the "typical" correlation coefficient is inversely proportional to this number. Thus, the presence of a large number of independent fluctuations along a beam will require long integration periods and accurate correlation equipment. A review of presently available analog correlation equipment suggests that the lower limit of correlation coefficients which can be reliably determined is of order 0.05, suggesting an upper limit on the number of integral scales of order twenty. Preliminary indications, using digital correlation methods indicate that this limit can be increased, but the actual limit remains to be established.

CHAPTER 5

CONCLUSIONS

The work reported here, together with the experimental data of (6) , would indicate that the "Crossed Beam Correlation Technique" does provide a new remote sensing method for the determination of the local properties of turbulent shear layers. Perhaps the most fundamental advantage of the method is that measurements can be made without the previous necessity of disturbing the flow field with the measuring instrument. Thus, measurements in supersonic and/or hot burning flows become a practical possibility. A second, in principle, advantage is the inherent flexibility of the method offered by the broad choice of the wavelength for the radiation employed. It is felt that future emploitation of this feature will permit investigations in depth to an extent which has not been possible previously with the more standard techniques.

The theoretical concept of the crossed beam correlation technique was reviewed in detail in Chapter 2 of this report. It is shown that, within the limits of certain well founded assumptions, suitable processing of the optical fluctuations can be employed to obtain local values of the turbulent intensity, integral turbulent scales, the frequency spectrum, convection velocity and rate of distortion of the turbulent pattern. The basic assumption involved is that correlated fluctuations exist only in a local region around the beam intersection point. It is perhaps an unfortunate feature of the method that the necessity to invoke this assumption puts the spatial resolution, in the plane containing the crossed beams, beyond the control of the experimenter. It is controlled by the flow field. On the other hand the stronger correlation existing between adjacent fluctuations offers a powerful weighting of fluctuations occurring in the immediate vicinity of the beam intersection point. Both our theoretical considerations and measurements of convection speed profiles (particularly those in supersonic flows indicate that a more than practically acceptable degree of spatial resolution is obtainable. Present results suggest that at least ten spatially resolved values of convection speed can be obtained across a single shear layer. Not only is this more than adequate for practical applications, but experience with point probes would suggest that at this degree of resolution the basic accuracy of statistical measurements might begin to set the resolution rather than a fundamental limitation of the crossed beam technique itself.

In contrast to resolution in the plane containing the crossed beams, the resolution in the remaining streamwise direction is controllable. It is set by the diameters of the beams, which is in turn a function of the fields of view of the detectors. To date measurements have been performed using diameters of order 1 mm, which appear more than adequate for the model flows investigated. In the event that even smaller models were employed or if a very fine turbulent structure were anticipated, beam diameter reduction would be desirable. The ultimate limit in this respect is set, in practice, by the brightness of the available light source, the magnitude of the optical fluctuations expected and diffraction phenomena. No general rules can be set down and it is felt that each situation must be reviewed on its own merits.

With regard to temporal resolution this is relatively unlimited with regard to detection problems, due to the very high frequency response available with present photo-detectors. The limitation in this respect is more likely to arise in data storage where it is necessary to store the signal from two independent photo-detectors without destroying their phase relationship. A currently available multiplex tape recorder system is capable of storing data with negligible relative phase distortion at frequencies up to 50 Kc/s. Previous experiments on a 2.15 in. diameter Mach 3.4 nozzle have indicated that maximum frequencies of interest are around 10-20 Kc/s. Thus, no limitations of temporal resolution are to be anticipated unless very small model flows were to be investigated, which would reduce the turbulent scales hence increasing frequency requirements. However, this limitation will apply irrespective of the measuring technique employed.

Chapter 3 of this report reviews the theory of the crossed beam technique with particular emphasis on the application of turbulence measurements to the estimation of the effects of the turbulent field on the local environment, the usual practical problem. It is shown that while the optical integration over the correlated region necessitated some assumptions to regain pointwise information, this integration is, in fact, advantageous here. The normal crossed beam measurement generates an area integral of the turbulent forcing function, while it is also demonstrated that the combination of a 'thick' and 'thin' beam can in principle be used to generate a volume integral. It is emphasized however that the practical problems involved in the latter have not been given any detailed consideration to date. However, it does appear that, while the estimation of turbulent forcing functions from point probe information necessitates large numbers of experiments and subsequent numerical integration of data, these functions can in principle be generated rather efficiently using the inherent integrating properties of the crossed beam method.

One further feature of the crossed beam method which justifies mention here is its ability to measure directly a very important and fundamental parameter of a turbulent field, the three-dimensional wave number spectrum. Direct measurement of this function has not been possible previously using conventional methods, while it is only in the singular case of isotropic turbulence that it can even be calculated from the measurable one-dimensional spectrum function. It is felt that subsequent comprehensive comparison of the one-dimensional function, obtained from hot wire measurements, with the three-dimensional function obtainable from the crossed beam method, will yield a great deal of insight into the structure and mechanisms which exist in turbulent shear layers.

Although up until this point this report has been principally concerned with the theoretical foundations of the crossed beam method, it would be unfair to end without a brief discussion of the difficulties involved in initially generating interpretable fluctuating signals at the two independent photodetectors. At the inception of our work, radiation from the vacuum ultra-violet region of the spectrum (circa 1850Å) was chosen. This spectral region is attractive, due to the fact that the required fluctuations are generated by the naturally present oxygen content of air driven facilities. However, it is felt that a brightness increase in available sources by a factor of at least one hundred is desirable for general detailed application. Even then the use of this spectral region involves a complex optical system. The necessity for special light sources, an evacuated optical system, precise monochromators and special optical windows are all complications which can be avoided by the use of visible radiation in conjunction with tracer techniques.

In considering suitable tracer materials there is little doubt that gaseous materials are to be preferred. Unfortunately all gases which absorb visible radiation are toxic and were therefore precluded for this work. Subsequent studies indicated a preference for solid tracer materials and it has been shown that aerosol particles of the correct size range ($\sim$0.5 microns) do offer a remarkably effective tracing method, concentration of order 1 part in 10^6 by volume or less being suitable for most applications. The major problems which arise here is the dissemination of sufficient tracer material in the required size range. Pretracing of the air storage supply is to be preferred since this allows maximum time for tracer dissemination. The injection of micron sized teflon particles initially suspended in liquid freon has been employed successfully. A study of metal oxide smokes also indicated these did offer a tracer of the desired size range, but a closer definition of the potential abrasion hazards of these materials is required.

Future consideration for alternative tracing methods should involve the employment of non-toxic gases in conjunction with infrared radiation. Carbon dioxide, either the natural air content or with artificial additions currently appears attractive. Unfortunately once again a move away from visible radiation will involve extra complications of the optical system. Finally, it should be mentioned that the use of the colored, toxic gases should not be overlooked although special precautions to contain the toxicity hazards will be necessary.

In spite of these difficulties, initial application of the crossed beam method involving the measurements in subsonic jet flows which are discussed in (6) are regarded as highly successful. The required degree of spatial resolution appears to have been obtained, while the close agreement with independent hot-wire investigations is very reassuring.

SUMMARY

Combining the theoretical work presented here with the experimental results of (6), it is submitted that the "Crossed Beam Correlation Technique" does offer a viable method for the determination of local properties of turbulent shear layers, while avoiding the necessity of disturbing the flow by the presence of an instrument.

Experimental studies indicate that a more than acceptable degree of spatial resolution is obtained, while temporal resolution is comparatively unlimited to the extent that currently available photo-detectors offer a frequency response far greater than is needed.

Advantages of the technique over more conventional methods involve far more direct estimates of turbulent forcing function and the capability of measuring the three-dimensional spectrum function.

From the practical viewpoint, it appears at present that future application of vacuum ultraviolet radiation to detect thermodynamic property changes in air flow would be considerably enhanced by the availability of brighter light sources although the method has been employed successfully. The use of solid tracers with visible radiation does offer a practical method of application, but gaseous tracers would be preferable. Carbon dioxide with infrared radiation should be explored as a possible method, while gases which absorb visible radiation would be preferable if protection from the attendant toxicity hazards can be provided.

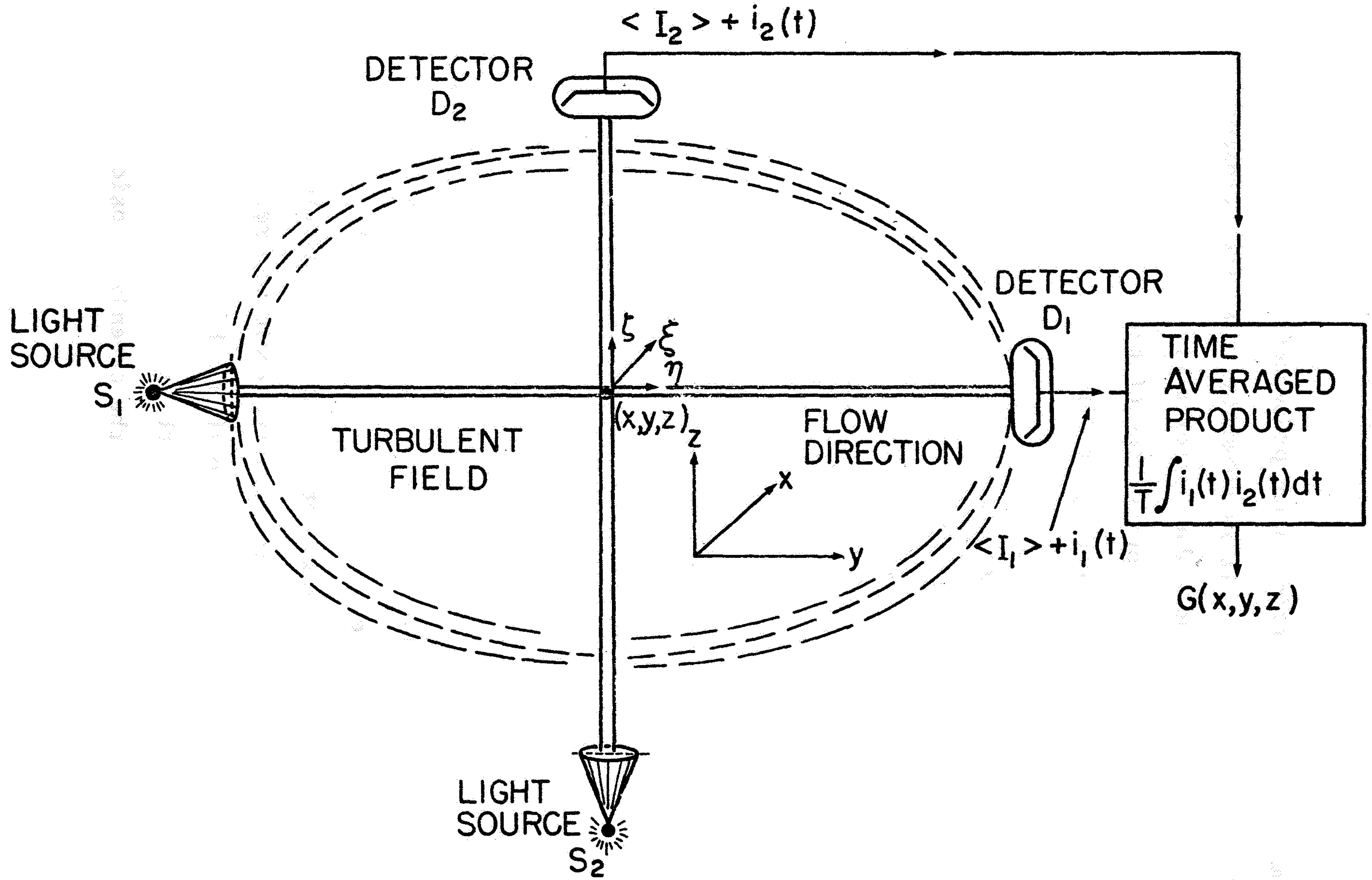


Fig. 1 SCHEMATIC DIAGRAM OF CROSSED BEAM CORRELATION PRINCIPLE

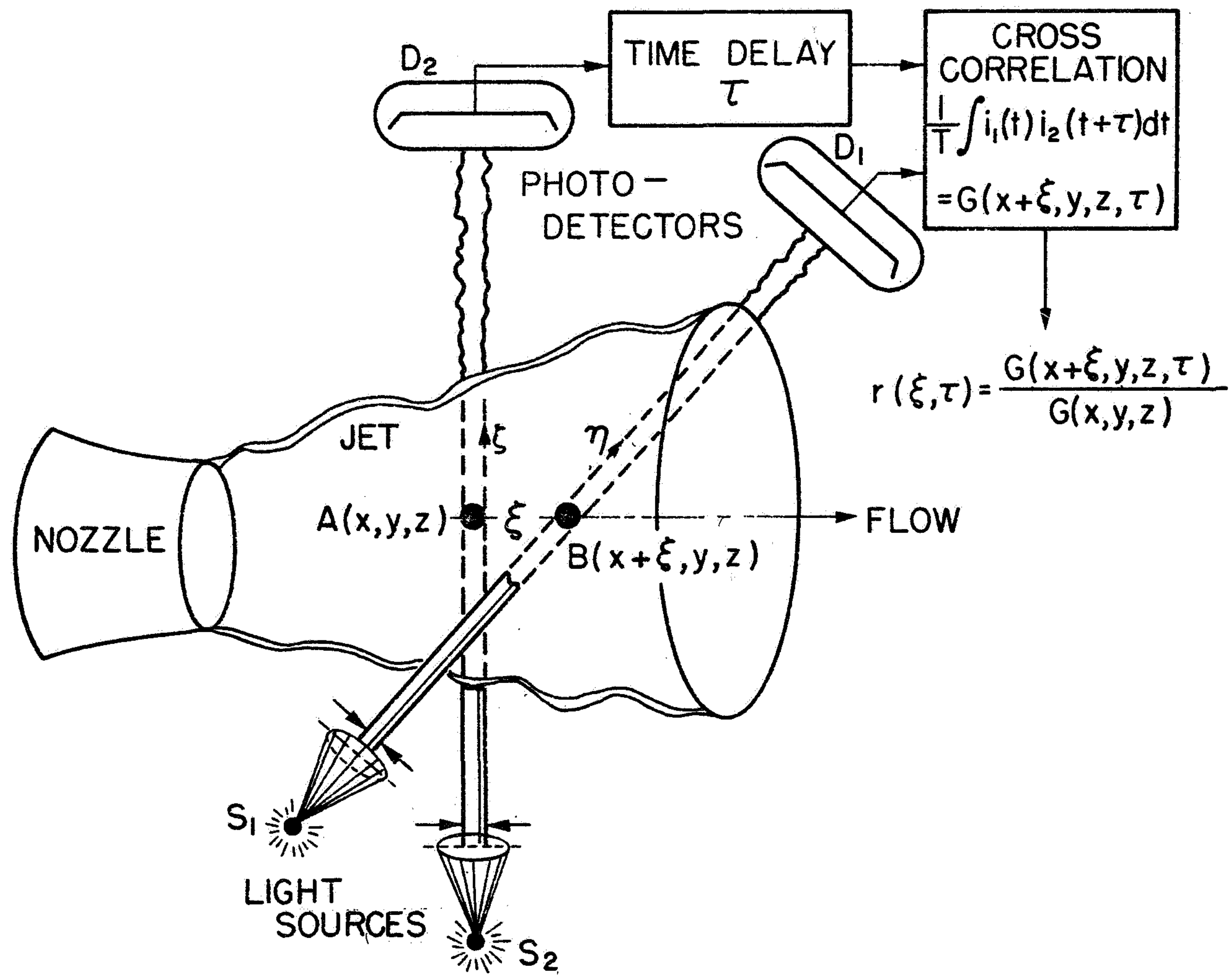


Fig. 2 SCHEMATIC DIAGRAM OF CROSSED BEAM OPERATION WITH DOWNSTREAM BEAM SEPARATION

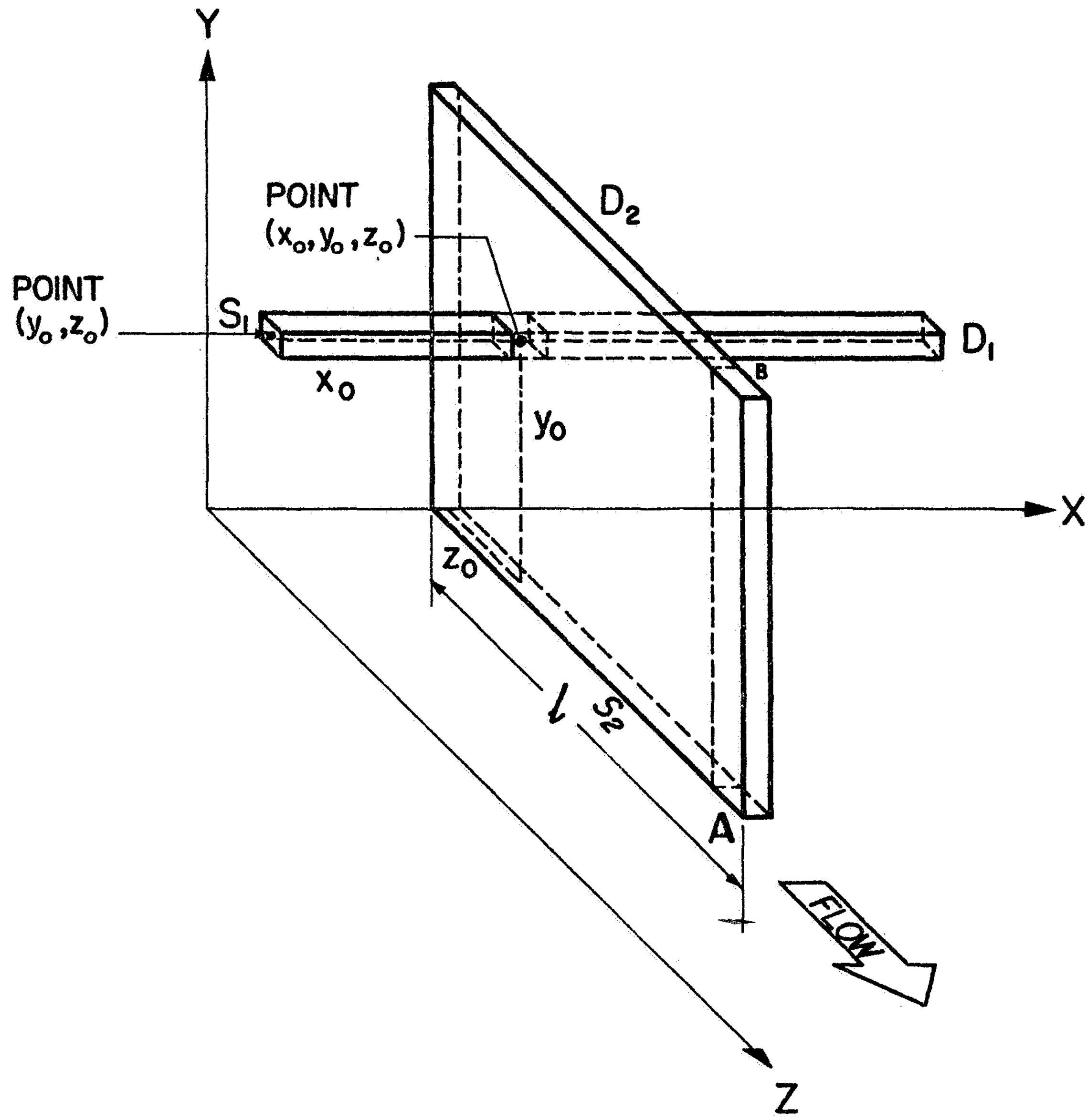
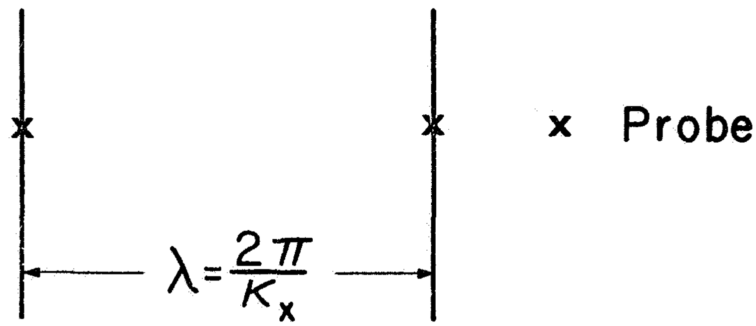
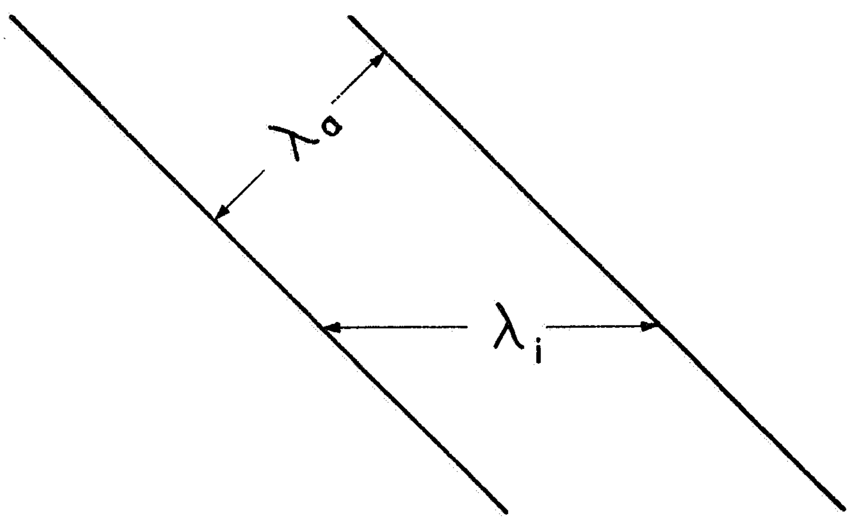


FIG. 3 INTERSECTION OF BEAM OF SMALL CROSS-SECTION WITH ONE OF LARGE CROSS-SECTION

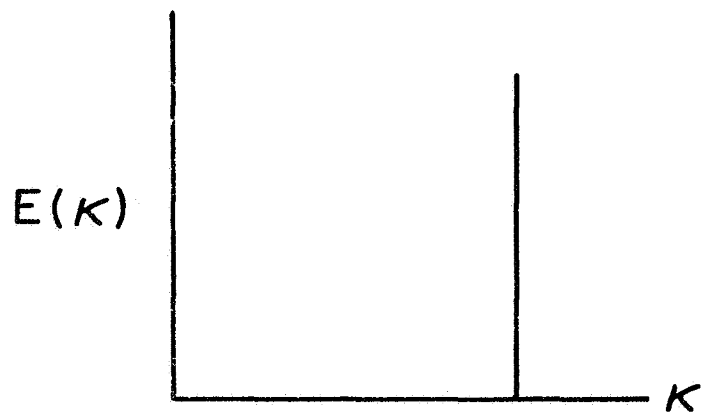


(a) Streamwise Disturbance

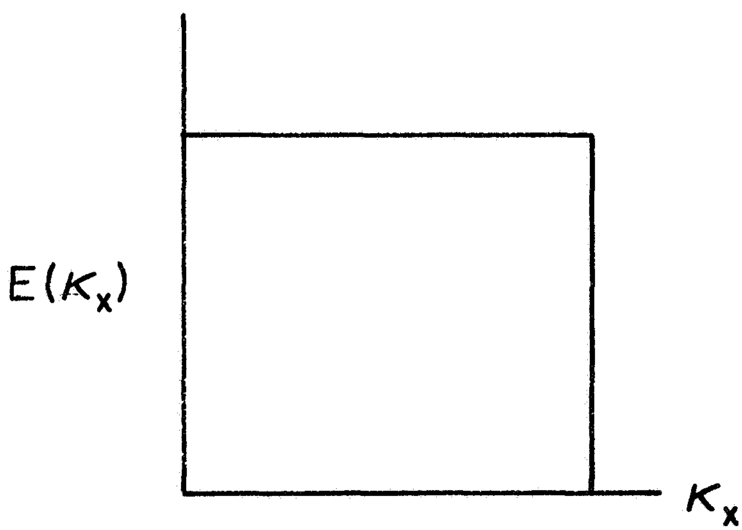


(b) Oblique Disturbance

Fig. 4 INTERPRETATION OF WAVE NUMBER SPECTRUM FOR TWO-POINT PROBES



(a) Three Dimensional



(b) One Dimensional

Fig. 5 WAVE NUMBER SPECTRA FOR FUNCTIONS

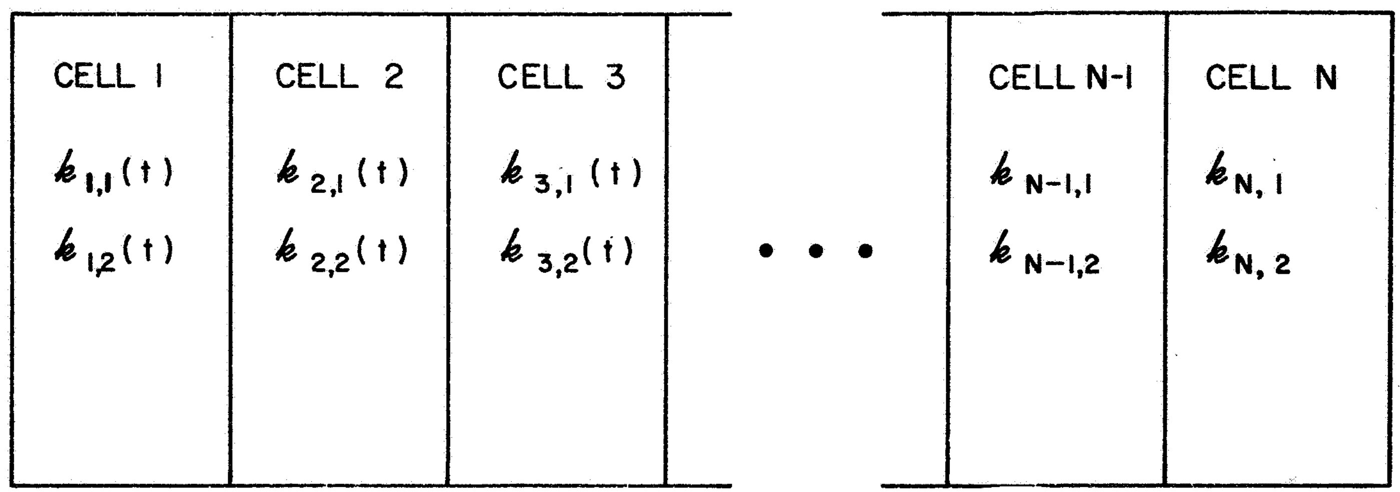


Fig. 6 ARTIFICIAL CROSSED-BEAM EXPERIMENT, EACH CELL CONTAINS AN ABSORBING SPECIES

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